Productivity and Debt in Relational Contracts

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Abstract

This paper studies how financial obligations constrain incentives, which generates productivity dynamics. A liquidity-constrained manager simultaneously repays a creditor and motivates a worker. If the manager cannot commit to output-contingent payments, then effort increases as the manager repays debts in a profit-maximizing equilibrium. The manager might defer worker compensation while repaying the creditor, in which case effort depends on both current and past debts. If the manager and worker can collude, which the creditor deter by threatening liquidation, then debt again has a lingering effect on the relational contract. Empirically, current and past debt increases are correlated with productivity decreases. JEL Codes: C73, D21, D86, G32.

Keywords: Relational Contract, Productivity, Debt

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1 Introduction

Managers borrow money in order to pursue new opportunities and grow their businesses, but even an attractive-looking investment is likely to fail unless employees work hard implementing it. Formal incentive schemes are not necessarily reliable in these environments because the manager can typically manipulate contractible measures of performance, for instance by hiding or diverting profits.\(^1\) A manager therefore faces a commitment problem with her employees: she earns a return on her investment only if she can credibly promise to reward their hard work (Bull (1987); Levin (2003)). An analogous commitment problem plagues the manager’s credit relationship, as creditors are willing to invest only if they believe that the manager will repay her debts rather than absconding with the proceeds of the project (Aghion and Bolton (1992); Hart (1995); Holmstrom and Tirole (1997)).

In this paper, we explore how a manager’s credit and employment relationships interact to shape productivity dynamics. We show that the manager’s financial obligations constrain her relational contracts with workers, since an indebted manager is less willing to follow through on promised incentives. Therefore, effort declines when a firm takes on debt and slowly recovers as that debt is repaid. In a profit-maximizing relational contract, debt both depresses contemporaneous productivity and potentially leads the manager to defer worker compensation, in which case debt continues to (temporarily) depress productivity even after it has been fully repaid. We complement this analysis with empirical evidence that changes in a firm’s debt are indeed negatively correlated with changes in both contemporaneous and future productivity.

As an example of how borrowing constrains relational incentives, consider Lincoln Electric’s decision to borrow in 1992 following devastatingly poor international performance. Lincoln Electric paid smaller discretionary bonuses to its U.S. workers to prioritize repaying this debt, a decision which threatened to undermine its strong relational incentive system. In a radical departure from the company’s cooperative worker-manager relations, Lincoln workers openly voiced their “disgruntlement” about small payments, while managers expressed fears that the entire incentive system might “unravel” if they did not pay bonuses (Feder (1994); Hastings (1999)). Reinforcing this point, Bae et al. (2011) shows that firms which

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\(^1\) Shleifer and Vishny (1997) discusses theory and evidence for these kinds of commitment problems in credit relationships, while Malcomson (2013) discusses similar commitment problems in agency relationships.
treat their employees fairly tend to maintain low debt ratios; Strebulaev and Yang (2013) documents that many U.S. firms have no or very little debt and that these firms perform well by a variety of measures; and Fahn et al. (2017) argues that firms which rely on relational contracts tend to have lower leverage.

In our model, a liquidity-constrained manager borrows money from a creditor to fund a project, then repeatedly motivates a worker to exert effort that creates profits in each period. The manager uses realized profits to repay the creditor and compensate the worker. If the manager cannot commit to repay the creditor or reward the worker—for example, because she can divert cash for her private benefit—then her promised payments are made credible by the threat that the worker might shirk or the creditor might liquidate the project if they are not paid.

The manager is willing to pay both her creditor and her worker because they will otherwise deny her future profits, which means that she will renege on both parties if she finds her aggregate promises too onerous. An indebted manager must promise large payments to the creditor, which limits the extent to which she can credibly reward the worker’s effort and therefore depresses output. In a profit-maximizing relational contract, the manager’s outstanding debt decreases over time, with concomitant increases in effort. These effort changes create firm-level productivity dynamics, since the strength of relational incentives are an omitted variable in empirical measures of total factor productivity.

Section 3 characterizes how borrowing generates effort dynamics in profit-maximizing equilibria. A key feature of such equilibria is that they entail multilateral punishments: the worker punishes the manager if she reneges on the creditor. Consequently, we show that the worker and creditor effectively act as a single player, in the sense that current productivity depends on the sum of their continuation payoffs. In equilibrium, the worker initially exerts little effort, which stochastically increases as the manager repays the creditor until it converges to a steady state that is independent of the initial debt. All payment schemes that support these productivity dynamics share the feature that the worker’s compensation is at least weakly backloaded. If this backloading is strict, then effort is negatively related to both current and past debt, with the implication that borrowing might depress firm productivity even after it has been repaid.

The profit-maximizing equilibrium has the feature that the worker punishes the manager “on behalf of” the creditor, which means that the creditor does not need to threaten
liquidation to induce repayment.\footnote{Such punishments can also be interpreted as “whistleblowing”; the worker communicates any deviation to the creditor, who punishes by liquidating the firm. Regardless of interpretation, the key feature of these equilibria is that, because the worker observes and so can condition actions on all outcomes, he never inefficiently punishes the manager on the path of an equilibrium with multilateral punishments.} In practice, the manager and worker might collude to undermine these multilateral punishments. To address this possibility, Section 4 defines a constraint that captures manager-worker collusion and then characterizes profit-maximizing equilibria subject to this constraint in a setting with binary effort. Unlike Section 3, worker and creditor do not act as a single player in these equilibria, so the creditor must rely on liquidation threats to induce repayment. These threats cannot condition on realized output and so liquidation might inefficiently occur on the equilibrium path so long as the creditor has not been repaid. Profit-maximizing equilibria pay the creditor first and defer worker compensation to minimize the probability of liquidation, which strengthens Section 3’s claim that debt can have both immediate and lingering effects on the manager’s promises to her worker.

Section 5 examines the empirical relationship between debt, wages, and productivity. Using data from Europe, we show that an increase in a firm’s leverage in either the current or previous year is correlated with a decrease in that firm’s current productivity. This result holds for multiple productivity measures and with either firm and year or industry-year fixed effects. We find suggestive evidence that this relationship is strongest in industries where employees contribute the most to production. We also show that increases in leverage are correlated with decreases in wages, which is consistent with our story for how debt affects the credibility of promised compensation. While these empirical results are by no means conclusive, they are consistent with our mechanism and suggest that within-firm debt dynamics are highly correlated with both wage and productivity dynamics.

Our model builds on the relational contracting literature (Bull (1987); MacLeod and Malcomson (1989); Levin (2003)) but applies these ideas to a manager who simultaneously manages relationships with both workers and creditors. Levin (2002) studies relational contracts with multiple agents but assumes transferable utility so that stationary equilibria are optimal. Recently, papers have considered dynamics that arise from liquidity constraints (Fong and Li (2017)), private information (Halac (2012); Malcomson (2016)), or subjective evaluation (Fuchs (2007)). Among these papers, Board (2011) studies multilateral contracts in the presence of liquidity constraint but does not consider financing relationships. An-
drews and Barron (2016) and Barron and Powell (2018), which study relational contracts under uncoordinated punishments, are related to our analysis of collusion in Section 4, but dynamics in those papers are driven by private monitoring rather than the possibility of collusion.

A growing literature studies how agency problems interact with financial constraints (Pagano and Vulpin (2008)). Like us, Hennessy and Livdan (2009) and Fahn et al. (2018) consider how borrowing constrains relational contracts, but both focus on stationary rather than dynamic equilibria. Michelacci and Quadrini (2009) considers wage dynamics that arise from a credit-constrained firm that “borrows” money from employees by deferring compensation. While we similarly consider the interaction of compensation and finance, our results consider productivity (in addition to wage) dynamics, and our mechanism differs by assuming limited commitment with the same punishment threat in both employment and financial relationships.\(^3\) Li and Matouschek (2013), Englmaier and Fahn (2017), and Fuchs et al. (2017) consider liquidity constraints or investments in relational contracts, but unlike our setting, these papers do not study the dynamics of agency and credit relationships.

A vast literature studies contracting frictions in financial relationships (Jensen and Meckling (1976); Myers (1977); Townsend (1979); Gale and Hellwig (1985); Stein (2003)). In particular, Aghion and Bolton (1992), Hart (1995), Holmstrom and Tirole (1997), and others focus on commitment problems in finance contracts. We build on this approach by analyzing how similar commitment problems also constrain within-firm incentives. More recently, He and Milbradt (2016), DeMarzo and He (2017), and the papers discussed in the next paragraph consider dynamic models of financing with limited commitment. Our analysis of collusion relates to Tirole (1986), Biais and Gollier (1997), and other static models of manager-worker collusion against an outside creditor.

Within this literature, our techniques are related to Thomas and Worrall (1994), which studies dynamics in foreign investment, and Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006), and DeMarzo and Fishman (2007), which study working capital dynamics in the relationship between a creditor and an entrepreneur. The dynamics in Section 3 are driven by constraints on the sum of the worker’s and the creditor’s equilibrium

\(^3\)Michelacci and Quadrini (2005) informally discusses limited commitment but does not have effort dynamics and assumes that employees and creditors can threaten to withhold different portions of future profit following a deviation.
continuation surplus that are similar to the constraints that lead to dynamics in Thomas and Worrall (1994) and Albuquerque and Hopenhayn (2004). Relative to those papers, we show that equilibrium dynamics can arise from effort choices and hence generate variation in profits even after controlling for working capital. We also argue that, since the manager might backload pay to the worker, repaid debts can affect current productivity. Both of these predictions are consistent with our empirical findings. Section 4 considers the possibility of manager-worker collusion, which departs further from these papers.

2 Model

Consider an infinite-horizon game with a manager ("she"), a creditor ("it"), and a worker ("he") who share a common discount factor $\delta \in [0, 1)$ and interact in periods $t \in \{0, 1, 2, \ldots\}$. The manager needs a loan of size $L > 0$ to start a project. To secure this funding, she offers a contract to the creditor that specifies a sequence of liquidation probabilities $l(\cdot) \in [0, 1]$ that can depend on the history of payments to the creditor. The payment to the creditor in period $t$ is denoted $r_t$, with corresponding contractible history $h_c^t = (r_0, \ldots, r_t)$.

The creditor accepts or rejects this formal contract. If it accepts, then it pays $L$, the project is funded, and the game continues; otherwise, the game ends and players earn 0. This decision is the creditor’s only action in the game, though the resulting contract $l(\cdot)$ determines the continuation game played by the manager and the worker.

If the project is funded, then the manager and the worker play a repeated game. All variables are publicly observed. We assume that the only contractible variables are a public randomization device realized at each stage of each period and repayments to the creditor, $(r_0, \ldots, r_t)$. In each period, the stage game is:

1. The worker chooses effort $a_t \in \mathbb{R}_+$.
2. A state of the world $\theta_t \in \{0, 1\}$ is realized, with $\Pr\{\theta_t = 1\} = p$.
3. Output $y_t = \theta_t a_t$ is realized.
4. The manager pays $b_t \geq 0$ and $r_t \geq 0$ to the worker and creditor, respectively, where $b_t + r_t \leq y_t$.
5. The project is liquidated with probability $l_t \equiv l(h_c^t)$.
If the project has not been liquidated, then the manager’s and worker’s period-\(t\) payoffs are
\[ \pi_t = y_t - b_t - r_t \]
and
\[ u_t = b_t - c(a_t), \]
respectively, where \(c(\cdot)\) is the worker’s cost of effort. We assume that \(c(\cdot)\) is non-negative, strictly increasing, strictly convex, with \(c(0) = c'(0) = 0\) and \(\lim_{a \to \infty} c'(a) = \infty\). The creditor earns \(r_t\) in period \(t\). Following liquidation, the game ends and all players earn 0.

Denote the manager’s, worker’s, and creditor’s normalized discounted continuation payoffs in period \(t\) by
\[ \Pi_t = \sum_{t'=t}^{\infty} \delta^{t'-t} (1-\delta) \pi_{t'}, \]
\[ U_t = \sum_{t'=t}^{\infty} \delta^{t'-t} (1-\delta) u_{t'}, \]
and
\[ K_t = \sum_{t'=t}^{\infty} \delta^{t'-t} (1-\delta) r_{t'}, \]
respectively. Let first-best effort \(a^{FB}\) satisfy \(c'(a^{FB}) = p\). We assume throughout that \(pa^{FB} - c(a^{FB}) > L\) so that under first-best effort, the project has strictly positive net-present value. Define \(h^t \in \mathcal{H}^t\) as a history in period \(t\). We consider profit-maximizing Subgame Perfect Equilibria (SPE), which maximize the manager’s \textit{ex ante} expected payoff among SPE.

It is worth spending a moment on the central contracting friction in this model, which is that the manager can refuse to pay \(r_t\) or \(b_t\) and suffer no worse punishment than losing her continuation payoff. Consistent with the literature on relational contracts, we model this situation by assuming that the formal contract can make neither payments nor liquidation conditional on \(y_t\), which means that the manager can always renege on promised payments. While many financial instruments condition on realized profits, managers typically face opportunities to divert those profits in non-contractible ways, for instance by stealing them directly or diverting them to privately beneficial but unprofitable ventures (see, e.g., Hart (1995); Holmstrom and Tirole (1997); Albuquerque and Hopenhayn (2004); and DeMarzo and Fishman (2007)). Our formulation is equivalent to one in which reported profits were contractible but the manager could mis-report those profits and pocket the difference.\(^4\)

Several features of the model are convenient but inessential for our basic intuition. For instance, we assume the manager keeps the entire output \(y_t\) if she reneges on promised payments, but our intuition continues to hold if she can consume only a (strictly positive) fraction of output after reneging. Further, players earn 0 upon liquidation, but allowing a positive scrap value would not substantively change the analysis. We assume that the creditor observes everything in the repeated game, but our results do not depend on what

\(^{4}\)For example, we could assume that reported profit \(\hat{y}_t\) is contractible but the manager can report \(\hat{y}_t < y_t\) and effectively steal \(y_t - \hat{y}_t\). If payments can condition on \(\hat{y}_t\) but not \(y_t\) and moreover must satisfy \(b_t + r_t \leq \hat{y}_t\), then our analysis would continue to hold.
the creditor observes, since it takes no actions in the repeated game.

A few other assumptions potentially impact the central intuition and so deserve further comment. For instance, apart from the initial loan, the manager can neither borrow nor save; we discuss this restriction further in Section 6. Moreover, effort is observable, which simplifies equilibrium incentives in order to more starkly highlight how financing constraints lead to effort dynamics. Finally, we assume that liquidation \( l(\cdot) \) can condition on repayments to the creditor but not on payments to the worker, which is irrelevant in Section 3 but is important in Section 4 because it allows the manager and worker to collude by “secretly” making side payments to one another.

3 Financing Constraints and Productivity

This section formulates the equilibrium payoff frontier as a dynamic program, considers two benchmarks that highlight the role of the manager’s commitment problem, and analyzes productivity and payment dynamics in profit-maximizing equilibria.

The worker or the creditor can each punish the manager equally harshly following a deviation, the former by choosing \( a_t = 0 \) in each period and the latter by liquidating the firm. While either of these punishments min-max the manager, the worker can condition his punishment on the realization of \( y_t \) and so punish the manager exactly when she reneges on a payment despite having money. Consequently, it is efficient for the worker to punish any deviation by the manager, including deviations in \( r_t \). We can therefore set \( l(\cdot) \equiv 0 \) and assume that the manager earns 0 following any deviation.

Our first goal is to write the dynamic program for a profit-maximizing equilibrium. Let

\[
E = \{(U, \Pi) | \exists K \in \mathbb{R} \text{ s.t. } \exists \text{ SPE with worker, manager, and creditor payoffs } (U, \Pi, K) \}
\]

be the set of the worker’s and the manager’s equilibrium continuation payoffs. Given \((U, \Pi) \in E\), define \( K(U, \Pi) \) as the creditor’s maximum continuation payoff if the worker and manager earn \( U \) and \( \Pi \), respectively. In a given period \( t \), denote \( b \geq 0 \) and \( r \geq 0 \) as the manager’s payments to the worker and creditor if \( \theta_t = 1 \) and note that \( b_t = r_t = 0 \) if \( \theta_t = 0 \). Let \((U_L, \Pi_L)\) and \((U_H, \Pi_H)\) be the worker’s and manager’s continuation surpluses from period \( t + 1 \) onwards if \( \theta_t = 0 \) or \( \theta_t = 1 \), respectively.
We can now write the conditions that must be satisfied for the worker and manager to respectively earn $U$ and $\Pi$ in equilibrium. First, play must satisfy promise-keeping constraints so that the worker and manager actually earn these payoffs,

\[
U = (1 - \delta)(pb - c(a)) + \delta(pU_H + (1 - p)U_L) \quad \text{and} \quad \Pi = (1 - \delta)p(a - b - r) + \delta(p\Pi_H + (1 - p)\Pi_L). \tag{PK-A, PK-P}
\]

Second, the worker must be willing to choose the equilibrium effort level $a \geq 0$. He earns $U$ surplus from following the equilibrium and no more than 0 surplus from deviating, so this incentive constraint is simply

\[
U \geq 0. \tag{IC}
\]

Third, the manager must be willing to pay $b$ and $r$ if $\theta = 1$. Since the manager earns 0 continuation profit if she reneges, she pays $r$ and $b$ if and only if these payments satisfy the dynamic enforcement constraint

\[
\delta\Pi_H \geq (1 - \delta)(b + r). \tag{DE}
\]

Fourth, we assume that the worker has no wealth and the manager does not borrow any further money from the creditor. Since $y = a$ if $\theta = 1$, $r$ and $b$ must satisfy the following limited liability constraints:

\[
r \geq 0; \quad b \geq 0; \quad r + b \leq a. \tag{LL}
\]

Finally, continuation payoffs must be attainable in equilibrium,

\[
(U_H, \Pi_H), (U_L, \Pi_L) \in E. \tag{CE}
\]

Given $(U, \Pi)$, actions and continuation payoffs in the equilibrium that maximizes the creditor’s payoff solve
\[ K(U, \Pi) \equiv \max_{r, b, a, U_H, U_L, \Pi_H, \Pi_L} \left( (1 - \delta)pr + \delta (pK(U_H, \Pi_H) + (1 - p)K(U_L, \Pi_L)) \right) \]

subject to \[(PK - A), (PK - P), (IC), (DE), (LL), (CE).\]

The project is funded if continuation utilities at the start of the game, \((U_0, \Pi_0) \in E\), satisfy \(K(U_0, \Pi_0) \geq L\). The rest of our analysis focuses on \(K(\cdot, \cdot)\), which corresponds to the equilibrium payoff frontier of this game.

Before turning to our main analysis, we consider two benchmarks. The first benchmark eliminates the manager's commitment problem entirely. In the commitment game, the formal contract specifies payments \(b_t\) and \(r_t\) as a function of the history of realized outputs, \(\{y_t\}_{t=0}^T\). The manager cannot renege on output-contingent payments in the formal contract. These payments therefore do not need to satisfy (DE), with the consequence that profit-maximizing play attains first-best.

**Proposition 1** In any profit-maximizing equilibrium of the commitment game, the project is funded and \(a_t = a^{FB}\) for all \(t \geq 0\).

**Proof:** See Appendix A.

If the manager can write an output-contingent formal contract, then she can commit to compensate the worker for \(a^{FB}\). She can also commit to repay the creditor any portion of the resulting surplus, which means that she can credibly promise to repay enough to fund the (positive net-present value) project.

Our second benchmark considers a setting in which the worker is essentially passive in order to highlight the role of the manager-worker relationship. In the bilateral game, the worker chooses \(a_t\), which the manager can either accept or reject. If she rejects, then effort and output equal 0 in that period; if she accepts, then effort equals \(a_t\) and the manager—but not the worker—incurs cost \(c(a_t)\). This benchmark essentially eliminates the need to motivate the worker, so we can ignore (IC) and set \(b_t = 0\) in each period. As in the baseline game, however, the worker can still punish the manager following a deviation by choosing \(a_t = 0\) in subsequent periods.

Payments to the creditor must satisfy (DE) in this benchmark, so the project might go unfunded if the manager cannot credibly promise to repay the creditor. So long as the
project is funded, however, effort equals first-best in each period.

**Proposition 2** In any profit-maximizing equilibrium of the bilateral game, the project is funded if and only if \( L \leq \frac{\delta p}{1-\delta+\delta p} \left( pa^{FB} - c(a^{FB}) \right) \). If the project is funded, then \( a_t = a^{FB} \) for all \( t \geq 0 \).

**Proof:** See Appendix A.

Suppose the project is funded in the bilateral game. Fixing payments to the creditor, increasing \( a_t \) increases the manager’s payoff by \( p - c'(a_t) \) and relaxes all constraints if \( a_t < a^{FB} \). Any profit-maximizing equilibrium will therefore implement first-best effort in each period. However, the manager is willing to repay the creditor only if she earns a strictly positive continuation surplus, which leads to a wedge between the value of the project and the amount that the creditor is willing to lend in equilibrium.

These two benchmarks suggest that if productivity dynamics arise at all, then they must arise from the manager’s commitment problems in both of her relationships. The first step in characterizing these dynamics is to prove that the total surplus that can be promised to creditor and worker, \( K(U, \Pi) + U \), is independent of how that surplus is split between these two players. Define

\[
\tilde{U}(\Pi) = \max \{ U | (U, \Pi) \in E \}
\]

as the worker’s maximum equilibrium continuation utility given \( \Pi \).

**Lemma 1** For any \( (U, \Pi) \in E \),

\[
K(U, \Pi) + U = \tilde{U}(\Pi).
\]

**Proof:** See Appendix A.

Lemma 1 says that the profit-maximizing equilibrium depends on only the sum of the worker’s and creditor’s continuation payoffs, which substantially simplifies the analysis of dynamics in this equilibrium. To prove this result, we show that given a sum \( K(U, \Pi) + U > 0 \), we can always find a period in which we can either decrease the creditor’s payment in order to increase the worker’s payment by the same amount, or decrease the worker’s payment without violating (IC) in order to increase the creditor’s payment. Consequently, we can increase
or decrease $U \geq 0$ without changing $K(U, \Pi) + U$. This argument builds on the intuition for multilateral punishments in Levin (2002), but our result holds even though manager and worker are both liquidity constrained and does not imply that profit-maximizing equilibria are stationary. Indeed, we will show that such equilibria typically entail dynamics.\

Given this result, we can characterize the equilibrium payoff frontier in terms of $\tilde{U}(\Pi)$. Define
\[
    a_{\text{max}} = \arg \max_{a \geq 0} \left\{ pa - c(a) \mid \delta(pa - c(a)) \geq \frac{(1 - \delta)c(a)}{p} \right\}
\] (2)
as the maximum effort that can be sustained in equilibrium if the creditor is paid nothing.

**Proposition 3** Consider on-path play in period $t$ of a profit-maximizing equilibrium, and let $\Pi$ be the manager’s continuation payoff at the start of that period. Then:

1. $a_t = a^*(\Pi)$, where
   \[
   a^*(\Pi) \equiv \min \left\{ a_{\text{max}}, \frac{(1 - \delta)(1 - p)}{(1 - \delta)p} \Pi \right\}.
   \]
2. If $a_t < a_{\text{max}}$, then $\Pi_L = \Pi < \Pi_H$ and so $a_{t+1} = a_t$ if $\theta_t = 0$ and $a_{t+1} > a_t$ if $\theta_t = 1$.
3. If $a_t = a_{\text{max}}$, then $a'_{t'} = a_{\text{max}}$ for all $t' > t$. Consequently,
   \[
   \lim_{t \to \infty} Pr\{a_t = a_{\text{max}}\} = 1.
   \]

**Proof:** See Appendix A.

Proposition 3 characterizes productivity dynamics in profit-maximizing equilibria. If the project is funded, then $U_0 = 0$ and $\Pi_0$ satisfies $K(0, \Pi_0) = L$ in $t = 0$. From there, effort and profit remain constant after $\theta_t = 0$ and increase after $\theta_t = 1$ until effort reaches $a_t = a_{\text{max}}$, at which point it stays at this steady state.

To prove this result, note that $\tilde{U}(\Pi)$ represents the payoff frontier if the creditor earns 0 continuation payoff. Along this frontier, we show that either $a = a^{FB}$ or (DE) must bind, since otherwise the manager could promise larger payments and motivate higher effort. Therefore, $b$ is pinned down by (DE); in particular, if $\tilde{U}(\Pi) = 0$, then $b = c(a)/p$ and $a =$

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5We can show that a version of Lemma 1 holds in more general settings with multiple workers and creditors, so long as effort is observable and players can jointly punish deviations. Consequently, similar productivity dynamics arise in richer environments.
If the worker earns strictly positive continuation utility, then he must not be working as hard as he would be willing to work given his expected compensation. Consequently, total surplus $\tilde{U}(\Pi) + \Pi$ is increasing in $\Pi$, which means the manager prefers frontloading payments in exchange for a higher continuation payoff so long as $\tilde{U}(\Pi_H) > 0$.

This intuition pins down $b$ and $\Pi_H$, so characterizing $\tilde{U}(\Pi)$ reduces to characterizing $\Pi_L$. Note that $\Pi_L$ and $U_L$ enter only (PK-A) and (PK-P). Since $\tilde{U}(\Pi)$ is concave, we can show that $\tilde{U}(\Pi_L) + \Pi_L = \tilde{U}(\Pi) + \Pi$ in any profit-maximizing equilibrium and so $\Pi_L = \Pi$, which is the final step to characterizing $\tilde{U}(\Pi)$. The shape of the payoff frontier $K(U, \Pi)$ then follows from Lemma 1.

The most important implication of Proposition 3 for our purposes is that, unless $L$ is small, effort starts below $a_{\text{max}}$ and only slowly and stochastically increases to this steady state. That is, the manager’s outstanding debt $K(U, \Pi)$ determines equilibrium effort and is shaped by the history of past repayments. Productivity dynamics arise because the amount owed to the creditor changes over time, with corresponding changes in the compensation that can be credibly promised to the worker.

Profit-maximizing equilibria also exhibits two other kinds of inefficiencies. The first inefficiency is familiar from many relational contracting models (e.g., Levin (2003)): unless players are quite patient, (DE) binds and so steady-state effort satisfies $a_{\text{max}} < a_{FB}$. The second inefficiency is familiar from papers on financing constraints (e.g., Holmstrom and Tirole (1997)): positive net-present value projects might go unfunded, since the manager must earn strictly positive profit to refrain from diverting funds (i.e., $K(0, 0) = 0$).

A variety of payment paths are consistent with profit-maximizing equilibrium. Our next result identifies two extreme equilibrium payment paths that repay the creditor either as quickly or as slowly as possible.

**Corollary 1** The following payment paths are both part of profit-maximizing equilibria.

1. Fastest repayment equilibrium: $r_t = y_t$ whenever $K(U_H, \Pi_H) > 0$. Once $K(U, \Pi) = 0$, $b_t = y_t$ whenever $U_H > 0$.

2. Slowest repayment equilibrium: whenever $K(0, \Pi_H) > 0$, $r_t = y_t - \frac{c(a_t)}{p}$ and $b_t = \frac{c(a_t)}{p}$, so that $U = 0$ in every period.
Proof: See Appendix A.

Corollary 1 follows from Lemma 1 and the proof of Proposition 3. Before productivity reaches its steady state, every profit-maximizing equilibrium entails the same total payment to the creditor and worker; the only question is how to split this payment between the two parties. To repay the creditor as quickly as possible, the manager defers the worker’s compensation until after the loan has been repaid. In contrast, the slowest repayment path exactly compensates the worker for his effort cost in each period.

If the worker’s compensation is deferred until after the creditor has been repaid, as it is in the fastest repayment equilibrium, then $U > 0$ in the first period that satisfies $K(U, \Pi) = 0$. If $U_H > 0$ in that period as well, then $a < a_{\text{max}}$ in that period. That is, productivity dynamics (temporarily) persist even after the creditor has been repaid.

Lemma 1 implies that in a profit-maximizing equilibrium, the worker and the creditor act as a single entity that loans the manager money and then exerts effort in each period. Consequently, effort dynamics in Proposition 3 resemble the working capital dynamics from Albuquerque and Hopenhayn (2004). There are three substantive differences. First, the worker cannot earn negative continuation utility, which implies that steady-state effort might remain inefficiently low, $a_{\text{max}} < a^{FB}$. Second, Albuquerque and Hopenhayn (2004) considers capital dynamics, while we explore effort and hence productivity dynamics. Finally, if borrowing leads to deferred worker compensation, then effort temporarily remains below the steady-state even after the creditor has been repaid. Section 5 explores the latter two differences empirically. In the next section, we turn to a setting in which Lemma 1 does not hold, which gives rise to the possibility of liquidation and hence further dynamics.

4 Collusion Between the Manager and Worker

Section 3 assumes that the worker punishes the manager if she reneges on the creditor. In practice, the manager and worker might collude by refusing to pay the creditor and splitting the proceeds between themselves. The creditor deters this collusion by threatening liquidation following non-repayment. But liquidation cannot be made contingent on realized output, so the resulting profit-maximizing equilibria entail new dynamics, including inefficient on-path liquidation.

To understand the consequences of collusion, we impose an equilibrium refinement on a
binary-effort version of our model. Consider the model from Section 2 with binary effort, so that \( a \in \{0, y\} \) and the worker’s cost equals 0 if \( a = 0 \) and \( c \) if \( a = y \). The following equilibrium refinement captures a notion of manager-worker collusion.

**Definition 1** A Subgame Perfect Equilibrium \( \sigma^* \) is a **truth-telling equilibrium** if at every on-path history \( h^t \) immediately before \( \theta_t \) is realized,

\[
E_{\sigma^*} \left[-(1 - \delta) r_t + \delta (\Pi_{t+1} + U_{t+1}) | h^t, \theta_t = 1 \right] \geq E_{\sigma^*} \left[ \delta (\Pi_{t+1} + U_{t+1}) | h^t, \theta_t = 0 \right].
\] (3)

The condition (3) requires that the sum of the manager’s and the worker’s equilibrium payoffs if \( \theta_t = 1 \) exceeds the sum of their payoffs if the manager instead refuses to pay \( r_t \) and continuation play is as if \( \theta_t = 0 \). Online Appendix C justifies this condition using the following thought experiment. Suppose that after the manager borrows money from the creditor, she can propose any continuation equilibrium to the worker. The manager can always propose an equilibrium in which the worker does not punish her for reneging on \( r_t \), which means the creditor cannot rely on the worker to punish on its behalf. Online Appendix C shows that (3) is a sufficient condition to ensure that the manager will nevertheless repay the creditor in equilibrium. This condition is related to the truth-telling constraints that appear in many entrepreneur-creditor models (e.g., Clementi and Hopenhayn (2006); DeMarzo and Fishman (2007)), with the key difference that (3) depends on the sum of the worker’s and the manager’s payoffs, rather than on the payoff of a single player.

A truth-telling equilibrium requires play to satisfy an additional **truth-telling** constraint,

\[
(1 - \delta) r \leq \delta (\Pi_H + U_H - \Pi_L - U_L).
\] (TT)

Let \( K^T(U, \Pi) \) be the solution to (1) subject to (PK-A)-(CE) and (TT), given \( a_t \in \{0, y\} \). Continuation payoffs always lie on this payoff frontier, so we can characterize equilibrium dynamics by characterizing \( K^T(\cdot) \).

**Proposition 4** Suppose

\[
\frac{c}{p} \leq \frac{\delta}{1 - \delta} (py - c).
\] (4)

Then:

1. If \( K^T(U_H, \Pi_H) > 0 \), \( b = 0 \) and \( r = y \).
2. There exists $\bar{\Pi} : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ such that $U + \Pi + K^T(U, \Pi)$ is strictly increasing in $\Pi$ for $\Pi < \bar{\Pi}(U + \Pi)$ and constant in $\Pi$ otherwise.

3. If $K^T(U, \Pi) > 0$, then liquidation occurs with positive probability in the continuation equilibrium.

Proof: See Appendix A.

The condition (4) ensures that the worker could be motivated to exert effort if the manager did not need to borrow money. The first part of Proposition 4 says that the manager initially uses the entire output to repay the creditor. During these periods, the worker is paid nothing and so is motivated by the promise of future compensation. The second part of Proposition 4 implies that the manager’s promises to the creditor and worker influence productivity, in the sense that total surplus depends on the profit retained by the manager, $\Pi$. The final part of the proposition says that inefficient liquidation occurs on the equilibrium path so long as the manager has not yet repaid the loan. In short: the creditor threatens liquidation to satisfy (TT). The manager minimizes the probability of liquidation by frontloading loan repayment and backloading worker compensation, so that the worker’s promised utility depends on past debts.

The proof of Proposition 4 is significantly more challenging than that of Proposition 3 because Lemma 1 does not hold in a truth-telling equilibrium. Consequently, equilibrium dynamics depend on the promised utilities to each of the creditor and the worker rather than on the sum of those utilities. We first argue that liquidating the project with positive probability is at least as efficient as setting $a = 0$, which means that we can set $a = y$ so long as the project has not yet been liquidated. Then we show that (TT) holds with $r > 0$ only if liquidation to occur with positive probability, which means that liquidation occurs with positive probability so long as the creditor has not been fully repaid. The manager pays the creditor before the worker in order to minimize the probability of liquidation, which implies that the worker’s continuation utility is strictly positive in the first period after the debt is repaid.

Proposition 4 says that every profit-maximizing truth-telling equilibrium entails deferred worker compensation, which strengthens Corollary 1’s result that there exist profit-maximizing equilibria with deferred compensation. Unlike Proposition 3, the firm is some-
times liquidated in this equilibrium, which means that play eventually reaches one of two steady states: either the loan is repaid and continuation surplus converges to $\delta \frac{\delta}{1-\delta} (py - c)$, or the firm is liquidated for failing to repay its debts.

Binary effort simplifies the analysis, but Online Appendix D shows that analogous properties hold with continuous effort. In particular, the manager backloads worker compensation whenever (TT) binds, in which case debt has a lingering effect on the worker’s continuation utility and hence on his effort.

5 Empirical Analysis

We can summarize the basic mechanism that drives both Propositions 3 and 4 in the following way: an indebted manager prioritizes repaying those debts, which means that she cannot credibly promise large rewards for effort, which leads to less effort. The manager might also also defer worker compensation, which leads to lower effort in the future as well.

This section documents robust correlations that are consistent with (though not conclusive of) this mechanism. While we cannot observe workers’ efforts directly, revenue total factor productivity is the residual from a regression of revenue on capital and labor. Effort is omitted from this regression, so effort dynamics will translate to productivity dynamics in the data. We show that increases in a firm’s past or current financial leverage are correlated with decreases in its total factor productivity, even after controlling for either firm and year or industry-year fixed effects. We find suggestive evidence that these effects are strongest for those firms that rely most heavily on labor inputs. We also show that increases in leverage are correlated with decreases in wages, which is consistent with a key part of our mechanism.

We use data from Amadeus, which includes yearly financial statements, measures of input, and revenue for a panel of European firms. Our sample covers 16 countries and almost 25 thousand public and private manufacturing firms, with up to 10 years of data per firm between 1985-2017. We measure borrowing by the ratio of non-current liabilities to total assets (book leverage), and we proxy for productivity using several measures of revenue total factor productivity (TFP-R). Our simplest specification, which measures TFP-R as the residual from an OLS regression of revenue on capital and labor, suffers from endogeneity concerns because firms’ input choices potentially respond to their unobserved productivity shocks. Therefore, we also calculate TFP-R using methods from Levinsohn and Petrin (2003).
and Gandhi et al. (2017), both of which use a firm’s intermediate input choices to address this concern. We use output revenue, fixed assets, total wage bill, and the cost of materials to proxy for output, capital, labor, and intermediate inputs, respectively. Our analysis restricts attention to industry-country pairs with at least 1,000 observations. Appendix B details our methodology.

Our first set of results regresses changes in productivity on changes in both current- and previous-year leverage:

$$\Delta TFP_{i,t} = \beta_1 \Delta Lev_{i,t} + \beta_2 \Delta Lev_{i,t-1} + \beta_3 X_{i,t} + \mu_i + \psi_t + \epsilon_{i,t}. \quad (5)$$

Here, $\Delta TFP_{i,t}$ is the change in TFP-R from years $t - 1$ to $t$ and $\Delta Lev_{i,t}$ is the same for financial leverage. We include both firm and year fixed effects, so our regression exploits within-firm changes in productivity and leverage while controlling for year-specific aggregate shocks. We also run an alternative specification that includes industry-year fixed effects in order to control for year-specific shocks to industry-level productivity. Additional controls $X_{i,t}$ include changes in number of employees and in total fixed assets. Our main results weight observations equally; Appendix B has regressions weighted by number of employees, total assets, or gross output.\(^6\) Standard errors are clustered by NACE2 industry code.

Table 1 shows that an increase in firm leverage in year $t$ or $t - 1$ is correlated with a decrease in year-$t$ productivity. This relationship is highly significant across measures of TFP-R and economically substantial: a one standard deviation increase in contemporary leverage is correlated with a decrease in TFP-R equal to 9-15% of the median within-firm standard deviation (see Appendix B for details on these calculations). The coefficient on the previous year’s leverage change is smaller but remains significant across specifications.

The results in Table 1 are consistent with our theory, but other mechanisms could also explain the negative relationship between debt and productivity. A natural alternative explanation is related to productivity shocks: firms with negative productivity shocks might have trouble funding operations and so take on more debt, which could explain the negative coefficient on contemporaneous leverage changes in (5). To explain the negative coefficient on leverage changes in year $t - 1$, however, this alternative mechanism would require changes in productivity to be persistent, so that decreases in productivity in the previous year imply

\(^6\)Results remain broadly similar, though $\Delta Lev_{i,t-1}$ is not consistently significant.
Table 1: Changes in TFP and Changes in Leverage

<table>
<thead>
<tr>
<th></th>
<th>(1) ΔTFP-OLS</th>
<th>(2) ΔTFP-OLS</th>
<th>(3) ΔTFP-LP</th>
<th>(4) ΔTFP-LP</th>
<th>(5) ΔTFP-GNR</th>
<th>(6) ΔTFP-GNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔLeverage</td>
<td>-0.432 (0.0472)</td>
<td>-0.417 (0.0412)</td>
<td>-0.160 (0.0229)</td>
<td>-0.150 (0.0203)</td>
<td>-0.475 (0.0749)</td>
<td>-0.409 (0.0650)</td>
</tr>
<tr>
<td>(t-1) ΔLeverage</td>
<td>-0.101 (0.0200)</td>
<td>-0.0858 (0.0137)</td>
<td>-0.0449 (0.0187)</td>
<td>-0.0452 (0.0183)</td>
<td>-0.0903 (0.0211)</td>
<td>-0.0650 (0.0108)</td>
</tr>
<tr>
<td>N</td>
<td>127703</td>
<td>127703</td>
<td>127703</td>
<td>127703</td>
<td>127703</td>
<td>127703</td>
</tr>
<tr>
<td>SD of Dep Var</td>
<td>0.12</td>
<td>0.12</td>
<td>0.08</td>
<td>0.08</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>SD of ΔLeverage</td>
<td>0.044</td>
<td>0.044</td>
<td>0.044</td>
<td>0.044</td>
<td>0.044</td>
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<tr>
<td>Firm FE</td>
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<td>-</td>
<td>Yes</td>
<td>-</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>-</td>
<td>Yes</td>
<td>-</td>
<td>Yes</td>
<td>-</td>
</tr>
<tr>
<td>Industry × Year FE</td>
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<td>Yes</td>
<td>-</td>
<td>Yes</td>
<td>-</td>
<td>Yes</td>
</tr>
</tbody>
</table>

All columns use an unbalanced panel with firm fixed effects and binary variables for each year. Additional controls include total fixed assets and number of employees. Dependent variable is change in TFP-R which is calculated based on OLS specification (columns 1 and 3), Levinsohn and Petrin (2003) method (LP, columns 2 and 4) and Gandhi et al. (2017) method (GNR, columns 3 and 6). In the bottom rows, standard deviation of dependent variable and of change in leverage is a within-firm value and median across all firms is presented. Standard errors are clustered on the industry level and shown in the parentheses.
further productivity decreases (and hence further leverage increases) in the current year. Appendix B considers this possibility by including lagged productivity changes, $\Delta TFP_{i,t-1}$, as a regressor in (5). Our results remain significant and many of the coefficients have larger magnitudes in this alternative specification.\(^7\)

Our mechanism suggests two further empirical implications. First, if debt constrains worker incentives, then leverage should have a more pronounced effect on productivity in industries that rely heavily on labor. We investigate this channel more directly by interacting leverage with a country-industry proxy for labor intensity, as measured by an indicator that equals 1 whenever the OLS-based coefficient of labor in the production function for a given industry is above the country-wide median.\(^8\) Table 2 offers suggestive evidence in favor of this prediction with significant negative coefficients on this interaction in 2 of 3 specifications (the p-value for the OLS coefficient is 0.11). Coefficients on the lagged interaction are typically negative but not significant.

Second, our mechanism requires that indebted managers are less able to credibly promise rewards for effort, which suggests that highly leveraged firms should pay low wages. We measure wages by dividing total compensation cost by the number of employees, then regress changes in this wage measure on changes in current and past leverage. Table 3 reports a negative correlation between changes in leverage and changes in wages after including either firm and year or industry-year fixed effects. This result is consistent with Benmelech et al. (2012), which finds that firms in financial distress negotiate substantially less generous labor contracts, and Matsa (2010), which argues that firms take on debt in order to improve their bargaining position versus organized labor. Our model builds on this connection to explore effort and hence productivity dynamics.\(^9\)

In our model, the worker’s effort affects contemporaneous profits but does not have a persistent effect on firm profitability. In practice, however, credibility problems could also

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\(^7\)Leverage might also be related to productivity shocks through changes in the firm’s equity valuation, for instance because the firm’s market value decreases when it receives a negative productivity shock. Such a relationship would generate a negative relationship between productivity and market leverage. However, we use book rather than market leverage because most of the firms in our sample are private.

\(^8\)We do not need to include this indicator as a separate regressor because it is absorbed by firm fixed effects.

\(^9\)Note that in our model, lower wages do not necessarily imply that the worker earns lower utility, since poorly compensated workers exert less effort. Therefore, our analysis is consistent with the idea that firms with higher leverage must compensate workers for bankruptcy risk.
Table 2: Effect for Labor Intensive Industries

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ΔTFP-OLS</td>
<td>ΔTFP-LP</td>
<td>ΔTFP-GNR</td>
</tr>
<tr>
<td>ΔLeverage</td>
<td>-0.352</td>
<td>-0.0846</td>
<td>-0.290</td>
</tr>
<tr>
<td></td>
<td>(0.0529)</td>
<td>(0.0279)</td>
<td>(0.0612)</td>
</tr>
<tr>
<td>ΔLeverage × Labor</td>
<td>-0.136</td>
<td>-0.129</td>
<td>-0.313</td>
</tr>
<tr>
<td>Intensive</td>
<td>(0.0803)</td>
<td>(0.0466)</td>
<td>(0.108)</td>
</tr>
<tr>
<td>ΔLeverage (t-1)</td>
<td>-0.103</td>
<td>-0.0283</td>
<td>-0.0884</td>
</tr>
<tr>
<td></td>
<td>(0.0214)</td>
<td>(0.0122)</td>
<td>(0.0196)</td>
</tr>
<tr>
<td>ΔLeverage (t-1) × Labor</td>
<td>0.00148</td>
<td>-0.0289</td>
<td>-0.00601</td>
</tr>
<tr>
<td>Intensive</td>
<td>(0.0442)</td>
<td>(0.0353)</td>
<td>(0.0390)</td>
</tr>
<tr>
<td>N</td>
<td>127703</td>
<td>127703</td>
<td>127703</td>
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<tr>
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</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Industry × Year FE</td>
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<td>-</td>
<td>-</td>
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</tbody>
</table>

All columns use an unbalanced panel with firm fixed effects and binary variables for each year. Additional controls include total fixed assets and number of employees. Dependent variable is change in TFP-R which is calculated based on OLS specification (column 1), Levinsohn and Petrin (2003) method (LP, column 2) and Gandhi et al. (2017) method (GNR, column 3). Standard errors are clustered on the industry level and shown in the parentheses.
Table 3: Changes in Average Wage and Changes in Leverage

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta L(Wage))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta L(Wage))</td>
<td>-0.109</td>
<td>-0.0832</td>
</tr>
<tr>
<td></td>
<td>(0.0144)</td>
<td>(0.0164)</td>
</tr>
<tr>
<td>(\Delta L(Wage)\ (t-1))</td>
<td>-0.0180</td>
<td>-0.0183</td>
</tr>
<tr>
<td></td>
<td>(0.0125)</td>
<td>(0.0104)</td>
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<tr>
<td>(N)</td>
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<td>127703</td>
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<td>Clustering</td>
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<td>Year FE</td>
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<td>-</td>
</tr>
<tr>
<td>Industry X Year FE</td>
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<td>Yes</td>
</tr>
</tbody>
</table>

All columns use an unbalanced panel with firm fixed effects and binary variables for each year. Additional controls include total fixed assets and number of employees. Dependent variable is change in log of average wage which is calculated as the ratio of total compensation costs and number of employees. Standard errors are clustered on the industry level and shown in the parentheses.

constrain hiring, retention, firm-specific human capital investments, and other choices that change a firm’s long-term growth. Matsa (2017) has a recent review of how a firm’s financial structure impacts its employment practices. Particularly related are recent empirical papers on the consequences of debt for employees’ hiring and retention (Brown and Matsa (2016); Baghai et al. (2016)). Relative to those papers, our theory suggests that debt might decrease productivity even if firms are far from bankruptcy. Our analysis also complements papers that study credit access and productivity, such as Manaresi and Pierri (2018). We argue that while the availability of credit might have a positive net effect on productivity, the act of borrowing itself can have lingering negative effects on effort.

6 Discussion

This section informally discusses several extensions and concludes.

**Saving and borrowing:** Apart from the initial loan, the manager neither borrows nor saves in our model. If the manager could borrow additional funds from the creditor, then she
could pay the worker more, which would decrease the worker’s promised continuation utility at the cost of increasing the creditor’s promised continuation utility. Without collusion, Lemma 1 suggests the sum of these utilities determines productivity, so that the manager would not benefit from further borrowing. With collusion, Proposition 4 suggests that borrowing would increase the probability of liquidation and so would be strictly inefficient. This logic suggests that borrowing would not affect equilibrium dynamics.

If the manager can save money and access those savings after she reneges, then she would still like to repay her obligations as quickly as possible and so we believe that savings would be irrelevant. In contrast, if the creditor or worker could prevent the manager from accessing savings, then accumulated savings would serve as a bond to deter the manager from deviating, which would potentially affect equilibrium dynamics.

**Investments with variable scale:** In some settings, the manager chooses the scale of her investment. Our model suggests two opposing distortions in the optimal investment scale. First, borrowing decreases effort, which should push the manager to pursue smaller, less expensive projects. On the other hand, investing in certain kinds of projects can ease commitment problems, particularly if a larger project implies that the manager has more to lose following a deviation (Klein and Leffler (1981); Halac (2015); Englmaier and Fahn (2017)). We can construct examples in which either of these two forces dominate, leading to either “insufficient” or “excessive” investment relative to first-best.

**Conclusion:** Our model highlights how borrowing influences the credibility of a firm’s promised incentives and thereby impacts effort, which is consistent with our empirical finding that changes in current or past debt are negatively correlated with changes in productivity. More generally, our model demonstrates how a firm’s financial obligations can constrain its incentive and organizational structure. Understanding these spillovers is crucial for understanding firm productivity, profitability, and growth.
References


A Omitted Proofs

A.1 Proof of Proposition 1

In the commitment game, suppose the manager offers a contract with
\[ r_t(y_t) = \frac{c(a^{FB})}{p} 1\{y_t \geq \frac{L}{p}\}, \]
\[ b_t(y_t) = \frac{c(a^{FB})}{p} 1\{a_t = a^{FB}\}, \]
and \( l_t = 0 \) in each \( t \geq 0 \). The creditor earns \( p r_t(y_t) = L \)
from this contract, while the worker earns \( p \frac{c(a^{FB})}{p} - c(a^{FB}) = 0 \) from choose \( a^{FB} \) and no
more than 0 from choosing any other effort. Therefore, this contract induces the creditor
to fund the project and the worker to choose \( a_t = a^{FB} \) in each period. The manager
earns \( pa^{FB} - c(a^{FB}) - L > 0 \), which is the maximum attainable surplus in any equilibrium.
Therefore, this contract is profit-maximizing and any profit-maximizing contract must fund
the project and induce \( a^{FB} \) in each period. ■

A.2 Proof of Proposition 2

In the unitary firm game, consider the following strategy: the manager offers a contract with
\( l_t = 0 \) in each \( t \), which the creditor accepts. In each \( t \) on the equilibrium path, \( a_t = a^{FB} \), the
manager accepts this effort, \( b_t = 0 \), and \( r_t = 1\{y_t = a^{FB}\} \frac{L}{p} \). Following any deviation, the
worker chooses \( a_t = 0 \) and the manager pays \( b_t = r_t = 0 \).

If \( L \leq \frac{\delta p}{1-\delta+\delta p} (pa^{FB} - c(a^{FB})) \), then this strategy is an equilibrium. The the manager
earns \( pa^{FB} - c(a^{FB}) - L > 0 \) and accepts \( a_t = a^{FB} \). She is willing to pay \( r_t = \frac{L}{p} \) so long as
\[ (1-\delta) \frac{L}{p} \leq \delta (pa^{FB} - c(a^{FB}) - L) \]
which is implied by \( L \leq \frac{\delta p}{1-\delta+\delta p} (pa^{FB} - c(a^{FB})) \). She is able to pay \( r_t = \frac{L}{p} \) because \( \frac{L}{p} \leq a^{FB} - \frac{c(a^{FB})}{p} < a^{FB} \). The manager earns the maximum attainable profit \( pa^{FB} - c(a^{FB}) - L > 0 \)
in this equilibrium, so if \( L \leq \frac{\delta p}{1-\delta+\delta p} (pa^{FB} - c(a^{FB})) \), then any profit-maximizing equilibrium
entails a funded project and \( a_t = a^{FB} \) in each period \( t \) on-path.

In any equilibrium, \( \Pi_t \leq pa^{FB} - c(a^{FB}) - K_t \) because \( U_t \geq 0 \). Therefore, (DE) requires
that at any \( h_t \) immediately following \( \theta_t = 1 \),
\[ (1-\delta)r_t \leq \delta \left(pa^{FB} - c(a^{FB}) - E[K_{t+1}|h_t]\right) . \] (6)
For each $t \geq 0$, define
\[ \mathcal{H}(t) \equiv \{ h^t | \theta_t = 1, \theta_{t'} = 0 \forall t' < t \} \]
as the set of histories such that $\theta_t = 1$ for the first time in period $t$. Then the project is funded only if
\[ \sum_{t=0}^{\infty} \delta^t (1 - p)^t p E [(1 - \delta) r_t + \delta K_{t+1} | \mathcal{H}(t)] \geq L. \]
Applying (6) to this expression yields
\[ \delta p \sum_{t=0}^{\infty} \delta^t (1 - p)^t S^{FB} \geq L \]
or \[ \delta p \frac{S^{FB}}{1 - \delta + \delta p} \geq L. \] So the project cannot be funded if $L > \frac{\delta p}{1 - \delta + \delta p} S^{FB}$. ■

A.3 Proof of Lemma 1

We first prove that $K(\tilde{U}(\Pi), \Pi) = 0$ for any $(U, \Pi) \in E$. Towards contradiction, suppose $K(\tilde{U}(\Pi), \Pi) > 0$ for some $(U, \Pi) \in E$. Then there exists some future period in which $r > 0$. In this period, consider the following perturbation: decrease $r$ to $\tilde{r} = r - \epsilon$ and increase $b$ to $\tilde{b} = b + \epsilon$. For sufficiently small $\epsilon > 0$, these perturbed payoffs continue to satisfy (DE) and (LL) in that and all previous periods because $r > 0$ and $r + b = \tilde{r} + \tilde{b}$, while (IC) is relaxed in that and all previous periods. But then $U < \tilde{U}(\Pi)$ and we obtain a contradiction. So $K(\tilde{U}(\Pi), \Pi) = 0$ for all $(U, \Pi) \in E$.

Next, we prove that $K(U, \Pi) = \tilde{U}(\Pi) - U$. If $U = \tilde{U}(\Pi)$, then this result holds by the previous argument. Note that if $\tilde{U}(\Pi) = 0$, then $U = \tilde{U}(\Pi)$. Suppose $0 \leq U < \tilde{U}(\Pi)$. Then we claim that $K(U, \Pi) > 0$. To prove this, consider the equilibrium in which the worker and manager earn $\tilde{U}(\Pi)$ and $\Pi$, respectively. We claim that there exists some period $t$ and history $h^t$ at the start of that period such that (i) $b_t > 0$ if demand is high in that period, (ii) $E[U_t | h^t] > 0$, and (iii) $E[U_{t'} | h^t] > 0$ for any $h^{t'}$ that precedes $h^t$.

To prove this, for each $\tau \geq 1$, define $\mathcal{H}^\tau$ as the set of histories such that $E[U | h^\tau] = 0$, but $E[U | h^t] > 0$ for all $h^t$ that precedes $h^\tau$. Define $\mathcal{H}^\infty$, with element $h^\infty \in \mathcal{H}^\infty$, as the set of infinite-horizon histories for which $E[U | h^\tau] > 0$ for every $h^\tau$ that precedes $h^\infty$. Then
\[ H^r \cap H^{r'} = \emptyset \text{ and } \bigcup_{\tau=0}^{\infty} H^\tau = H, \] so the worker’s payoff can be written \[ U = \sum_{\tau=0}^{\infty} \Pr_{\sigma^\tau} \{ H^\tau \} \left( E \left[ (1 - \delta) \sum_{t=0}^{\tau-1} \delta^t (b_t - c(a_t)) | H^\tau \right] \right). \]

Since \( U > 0 \) and \( c(a_t) \geq 0 \), it cannot be that \( b_t \equiv 0 \) in this expression. That is, there must exist some \( \tau, h^\tau \in H^r \), and \( h^t \) that precedes \( h^\tau \) such that \( b_t > 0 \) with positive probability at \( h^t \). By definition, \( E[U_t|h^t] > 0 \) and \( E[U_{t'}|h^{t'}] > 0 \) for any \( h^{t'} \) that precedes \( h^t \).

Consider decreasing \( b_t > 0 \) at this \( h^t \) and increasing \( r_t \) by the same amount. This perturbation satisfies (DE) and (LL) because \( b_t > 0 \) and \( b_t + r_t \) is constant, while (IC) is slack at \( h^t \) and every predecessor history and so continues to hold for a sufficiently small perturbation. Since \( K + U \) remains constant in this perturbation, which can be performed for any \( U > 0 \), \( K(U, \Pi) > 0 \) whenever \( 0 < U < \bar{U}(\Pi) \). If \( U = 0 \), then \( K(U, \Pi) > 0 \), and we can decrease \( r \) in some period holding \( b + r \) fixed to transfer utility from the creditor to the worker. Therefore, \( K(U, \Pi) + U \) is constant in \( \Pi \), and so \( K(U, \Pi) = \bar{U}(\Pi) - U \). ■

### A.4 Proof of Proposition 3

By Lemma 1, \( \bar{U}(\Pi) \) characterizes equilibrium payoffs. Note that the public randomization device implies that \( \bar{U}(\Pi) \) is concave. Let \( \Pi_{\text{max}} \) be the maximum \( \Pi \) such that \( (U, \Pi) \in E \), and note that when \( U = \bar{U}(\Pi) \), \( r_t = 0 \) in every period \( t \) on the equilibrium path.

**Part 1: \( \bar{U}(\Pi) + \Pi \) is increasing in \( \Pi \), and strictly so unless \( \bar{U}(\Pi) + \Pi = S^{FB} \).** First, we claim that \( \bar{U}(\Pi) + \Pi \) is weakly increasing in \( \Pi \).

For any \( \Pi \) such that \( \bar{U}(\Pi) > 0 \), the proof of Lemma 1 implies that there exists some history \( h^t \) such that \( b_t > 0 \) with positive probability at \( h^t \), \( E[U_t|h^t] > 0 \), and \( E[U_{t'}|h^{t'}] > 0 \) for any \( h^{t'} \) that precedes \( h^t \). Consider decreasing \( b_t \). Both \( \text{(DE)} \) and \( \text{(LL)} \) are relaxed by this change, while \( \text{(IC)} \) continues to hold in every previous period because it was previously slack. So this perturbation is consistent with equilibrium, increases \( \Pi \), decreases \( U \), and holds total surplus fixed. Hence, total surplus \( \bar{U}(\Pi) + \Pi \) must be weakly increasing in \( \Pi \).

Since \( \bar{U}(\cdot) \) is concave, there exists a (possibly corner) \( \bar{\Pi} \) such that \( \bar{U}(\Pi) + \Pi \) is strictly increasing for \( \Pi < \bar{\Pi} \). We argue that \( \text{(DE)} \) binds whenever \( a < a^{FB} \). Suppose not; then we can perturb the equilibrium by increasing \( b \) and \( a \) so that \( pb - c(a) \) remains constant. This
perturbation satisfies (IC) by construction, (DE) by assumption, and (LL) because \( c'(a) < p \) for \( a < a^{FB} \).

Now, consider two cases. First, suppose

\[
(1 - \delta) \frac{c(a^{FB})}{p} \leq \delta(p a^{FB} - c(a^{FB})). \tag{7}
\]

Then we claim that \( \tilde{U}(\bar{\Pi}) + \bar{\Pi} = S^{FB} \). Consider the stationary strategy profile that sets \( a = a^{FB} \text{ and } b = \frac{c(a^{FB})}{p} \). This strategy profile clearly satisfies (IC) and (LL), and also satisfies (DE) because (7) holds. Therefore, \((0, S^{FB}) \in E \) and hence \( \tilde{U}(\bar{\Pi}) + \bar{\Pi} = S^{FB} \).

Suppose instead that (7) does not hold. Then we claim that \( \tilde{U}(\bar{\Pi}) = 0 \). Suppose towards contradiction that \( \tilde{U}(\Pi) > 0 \). If \( \Pi_L < \bar{\Pi} \), then we can perturb the equilibrium by increasing \( \Pi_L \), which would strictly increase total surplus by definition of \( \bar{\Pi} \) and continue to satisfy (IC) because \( \tilde{U}(\bar{\Pi}) > 0 \). Similarly if \( \Pi_H < \bar{\Pi} \); hence, \( \Pi_L, \Pi_H \geq \bar{\Pi} \). But then \( \tilde{U}(\bar{\Pi}) + \bar{\Pi} = (p \tilde{a} - c(\tilde{a})) \), where \( \tilde{a} \) is the effort induced in any equilibrium with \( \Pi \geq \bar{\Pi} \) and \( U = \tilde{U}(\Pi) \).

Hence, any such payoffs can be sustained in a stationary equilibrium with effort \( \tilde{a} \). But then \( \tilde{a} < a^{FB} \), since if \( \tilde{a} = a^{FB} \), (DE) requires (7) to hold. So (DE) binds for all \( \Pi \geq \bar{\Pi} \) and \( U = \tilde{U}(\Pi) \). Since \( \tilde{U}(\Pi) > 0 \), there exists an interval of feasible \( \Pi > \bar{\Pi} \). In particular, for the \( \Pi \) such that \( \tilde{U}(\Pi) = 0 \), \((1 - \delta) \frac{c(\bar{a})}{p} = \delta \Pi \). But \((1 - \delta) \frac{c(\bar{a})}{p} \geq \delta \bar{\Pi} \) and \( \bar{\Pi} < \Pi \), obtaining contradiction.

**Part 2: for any** \( \Pi < \bar{\Pi}, \Pi_L = \Pi < \Pi_H \). Denote the right-hand and left-hand derivatives of \( \tilde{U} \) by \( \partial_+ \tilde{U} \) and \( \partial_- \tilde{U} \), respectively, and note that \( \partial_- \tilde{U}(\Pi) \geq \partial_+ \tilde{U}(\Pi) \) because \( \tilde{U} \) is concave.

We first claim that if \( \Pi < \bar{\Pi} \), then \( \partial_+ \tilde{U}(\Pi_L) \leq \partial_+ \tilde{U}(\Pi) \leq \partial_- \tilde{U}(\Pi) \leq \partial_- \tilde{U}(\Pi_L) \). Suppose \( \partial_+ \tilde{U}(\Pi_L) > \partial_- \tilde{U}(\Pi) \). Since \( \Pi < \bar{\Pi} \), \( \tilde{U}(\Pi) > 0 \); therefore, we can perturb the equilibrium by increasing \( \Pi_L \) without violating (IC). Doing so increases \( \Pi \) at rate \((1 - p) \delta \) and decreases \( U \) at rate \((1 - p) \delta \partial_+ \tilde{U}(\Pi_L) \). But this perturbation remains and equilibrium and hence \( \partial_+ \tilde{U}(\Pi) \geq \frac{(1 - p) \delta \partial_+ \tilde{U}(\Pi_L)}{(1 - p) \delta} \), which contradicts our assumption. Similarly, if \( \partial_- \tilde{U}(\Pi_L) < \partial_- \tilde{U}(\Pi) \), then we can prove contradiction by decreasing \( \Pi_L \), which does not violate any constraint.

Next, we argue that \( \partial_+ \tilde{U}(\cdot) \) is strictly decreasing for all \( \Pi < \bar{\Pi} \), so that \( \partial_+ \tilde{U}(\Pi_L) \leq \partial_+ \tilde{U}(\Pi) \) and \( \partial_- \tilde{U}(\Pi) \leq \partial_- \tilde{U}(\Pi_L) \) imply that \( \Pi = \Pi_L \). Towards contradiction, suppose that \( \tilde{U}(\cdot) \) is linear on an interval \( \Pi^A < \Pi^B \). Let \((b^i, a^i, \Pi^i_L, \Pi^i_H) \) be the bonus, action, and continuation payoffs associated with \( \Pi^i, i \in \{A, B\} \). Then for any \( \alpha \in (0, 1) \), \( b = \)
\( \alpha b^A + (1 - \alpha)b^B, a = \alpha a^A + (1 - \alpha)a^B, \Pi_L = \alpha \Pi_L^A + (1 - \alpha)\Pi_L^B, \text{and } \Pi_H = \alpha \Pi_H^A + (1 - \alpha)\Pi_H^B \) are an equilibrium. If \( a^A \neq a^B \), then the worker’s payoff from this convex combination is strictly larger than \( \alpha \tilde{U}(\Pi^A) + (1 - \alpha)\tilde{U}(\Pi^B) \) because \( c(\cdot) \) is strictly convex. Hence, \( a^A = a^B \).

Now, (DE) must bind for any \( \Pi < \bar{\Pi} \). Suppose it does not, so \( a^A = a^B \geq a^{FB} \). Then the upper bound of (LL) binds, since otherwise we could increase \( b \), which increases \( U \) while holding total surplus fixed and therefore contradicts \( \Pi < \bar{\Pi} \). Moreover, \( \Pi_H \leq \Pi \), since otherwise we could decrease it without affecting total surplus, which similarly contradicts \( \Pi < \bar{\Pi} \). But then

\[
(1 - \delta)b = (1 - \delta)a \geq (1 - \delta)a^{FB} > \delta \bar{\Pi} \geq \delta \Pi_H,
\]

where the equality follows from binding (LL), the first weak inequality follows from \( a \geq a^{FB} \), the strict inequality follows by (7), and the second weak inequality follows because \( \Pi_H \leq \bar{\Pi} \). But then (DE) is violated; contradiction.

Since (DE) binds,

\[
\Pi^A = (1 - \delta)y + \delta(1 - p)\Pi_L^A.
\]

Furthermore, \( \Pi_L^A \geq \Pi^A \), since if \( \Pi_L^A < \Pi^A \) then \( \partial_x \tilde{U}(\Pi_L^A) > \partial_x \tilde{U}(\Pi^A) \), since \( \Pi^A \) is the left endpoint of the linear segment. By an analogous argument, \( \Pi_L^B \leq \Pi^B \).

If \( \Pi_L^B < \Pi^B \), we can increase \( \Pi_L^B \), which contradicts that \( \Pi^B \) is the right endpoint of the linear segment. So \( \Pi_L^B = \Pi^B \), and by a similar argument, \( \Pi_L^A = \Pi^A \). But then

\[
\Pi^A = \frac{1 - \delta}{1 - \delta(1 - p)}a^A = \frac{1 - \delta}{1 - \delta(1 - p)}a^B = \Pi^B,
\]

contradicting \( \Pi^A < \Pi^B \).

We have shown that \( \partial_x \tilde{U}(\cdot) \) is strictly decreasing for all \( \Pi < \bar{\Pi} \). Therefore, \( \Pi_L = \Pi \) on this range.

**Part 3: effort, profit, and repayment dynamics.** We argue that in the equilibrium yielding payoffs \((\tilde{U}(\Pi), \Pi) \in E, a = a^*(\Pi) \) as defined in the statement of Proposition 3. Indeed, suppose (7) holds. Then \( a^*(\Pi) = a^{FB} \) for any \( \Pi \geq \bar{\Pi} \) and so \( a = a^*(\Pi) \) follows immediately. Otherwise, we argued in Part 2 that (DE) binds. Therefore,

\[
\Pi = (1 - \delta)pa + \delta(1 - p)\Pi_L,
\]
Since $\Pi = \Pi_L < \bar{\Pi}$, $a = \frac{1-\delta(1-p)}{(1-\delta)p}\Pi < a^FB$ and so $a = a^*(\Pi)$.

For profit dynamics, we argue that $\Pi_H > \Pi$ whenever $\Pi < \bar{\Pi}$. Suppose not; then $\Pi_H < \bar{\Pi}$. But then the upper bound of (LL) must bind. Otherwise, we could increase $b$ and increase $\Pi_H$ so that $(1-\delta)b + \delta\Pi_H$ is constant, which satisfies all constraints because $\bar{U}(\Pi_H) + \Pi_H$ is strictly increasing in this range. Therefore,

$$\Pi = \delta p\Pi_H + \delta(1-p)\Pi_L \leq \delta\Pi,$$

since $\Pi_H \leq \Pi$ by assumption and $\Pi_L = \Pi$ by Part 2.

Finally, we argue that $a_t$ converges to $a_{max}$ as $t \to \infty$ with probability 1 in any profit-maximizing equilibrium. Since $\Pi_L = \Pi$ and high output is realized with probability $p > 0$ in each period, it suffices to show that with probability 1, any profit-maximizing equilibrium reaches a period in which $\Pi_H \geq \bar{\Pi}$. So long as $\Pi_H < \bar{\Pi}$, (LL) binds and so

$$\Pi_H - \Pi = \frac{1 - \delta}{\delta p}\Pi.$$

For any $K > 0$, $\Pi > 0$ in the first period of any equilibrium in which the project is funded. Therefore, the manager’s profit increases by an amount that is bounded away from 0 every time high output is realized (and remains constant following low output). So $\Pi_H \geq \bar{\Pi}$ after a finite number of high outputs, which happen with probability 1 as $t \to \infty$. ■

A.5 Proof of Corollary 1

Fastest Repayment Path: We first claim that this payment path repays the creditor as quickly as possible. Indeed, for any payment path such that $b > 0$ in a period for which $K(U_H, \Pi_H) > 0$, we can decrease $b$ and increase $U_H$ so that $(1-\delta)b + \delta U_H$ remains constant, and increase $r$ so that $b + r$ remains constant. Lemma 1 implies that both worker and creditor earn the same expected surplus following this perturbation. However, perturbing the equilibrium in this way decreases $K(U_H, \Pi_H)$ and therefore repays the creditor faster. Since both (DE) and the upper bound of (LL) bind whenever $\Pi_H < \bar{\Pi}$ in any profit-maximizing equilibrium, the specified payment path is consistent with a profit-maximizing equilibrium.
**Slowest Repayment Path:** This payment path sets \( U = 0 \) in each period, which clearly maximizes \( K(U, \Pi) = \hat{U}(\Pi) - U \) in that period and so repays the creditor as slowly as possible. The proof of Proposition 3 shows that both (DE) and the upper bound of (LL) bind whenever \( \Pi_H < \bar{\Pi} \). This payment path also satisfies the lower bound of (LL), since \( b = \frac{c(a)}{p} \geq 0 \) and \( r = a - b \geq 0 \) because \( pa - c(a) \geq 0 \) for any \( a \leq a_{\text{max}} \). Therefore, these payments are consistent with a profit-maximizing equilibrium. ■

### A.6 Proof of Proposition 4

Analogous to the set \( E \), define

\[
E^T \equiv \{(U, \Pi) | \exists K \geq 0 \text{ such that } (U, \Pi, K) \text{ are truth-telling equilibrium payoffs}\}.
\]

Define the problem (P) as maximizing (1) subject to (PK-A)-(LL), \((U_H, \Pi_H) \in E^T \) and \((U_L, \Pi_L) \in E^T \), and (TT), with the restriction to \( a \in \{0, y\} \). Let \( K^T(U, \Pi) \) be the value function for (P).

This proof characterizes \( K^T(\cdot) \) with a series of lemmas, then uses that characterization to prove Proposition 4.

**Lemma 2**

Define

\[
E_1 \equiv \{(U, \Pi) \in E^T | \exists \text{ a solution to (P) with } a = y\} \subseteq E^T,
\]

and let \( E_0 \equiv E^T \setminus E_1 \). Then \((0, 0) \in E_0 \) and \( K^T(0, 0) = 0 \), so that \((0, 0) \) can be supported by liquidating the firm. Moreover, any \((U, \Pi) \in E_0 \) can be implemented by randomizing between \((0, 0) \) and some \((U', \Pi') \in E_1 \).

**Proof of Lemma 2**

First, note that \((0, 0) \in E_0 \), since otherwise (PK-P) would be violated. Consider any \((U, \Pi) \) that can be implemented with \( a = 0 \). Then \( U_L = \frac{U}{\delta}, \Pi_L = \frac{\Pi}{\delta} \), and consequently \( K^T(U, \Pi) = \delta K^T \left( \frac{U}{\delta}, \frac{\Pi}{\delta} \right) \). In particular, \( K^T(0, 0) = \delta K^T(0, 0) \) and so \( K^T(0, 0) = 0 \), which can be attained through liquidation.

Consider \((U, \Pi) \in E_0 \) with \( (U, \Pi) \neq (0, 0) \). If \((U, \Pi) \) can be implemented with \( a = 0 \), then \( K^T(U, \Pi) = \delta K^T \left( \frac{U}{\delta}, \frac{\Pi}{\delta} \right) = \delta K^T \left( \frac{U}{\delta}, \frac{\Pi}{\delta} \right) + (1 - \delta)K^T(0, 0) \leq K^T(U, \Pi) \), where the first
equality follows by the argument above, the second follows because $K^T(0,0) = 0$, and the inequality holds because $K^T$ is concave. So $K^T(U, \Pi)$ is linear between $(0,0)$ and $(\frac{U}{\delta}, \frac{\Pi}{\delta})$ for any $(U, \Pi) \in E_0$. Let $(U', \Pi')$ be the right endpoint of this linear segment, and note that $(U', \Pi') \in E_1$. Therefore, any $(U, \Pi) \in E_0$ can be implemented by randomizing between liquidation and some $(U', \Pi') \in E_1$.

Finally, if $(U, \Pi) \in E_0$ can be implemented with $a$ such that $0 < \Pr \{a = y\} < 1$, then $(U, \Pi)$ can also be implemented by randomizing between a point in $E_1$ and a point that can be implemented with $a = 0$. So by the above argument, such $(U, \Pi)$ can also be implemented by randomizing between continuation and liquidation. ■

Now, define

$$
\Pi_{\text{max}} = py - c; \\
\Pi_f = \frac{p(1-\delta)y}{1-(1-p)\delta}; \\
\bar{U}^T(\Pi) = \max \{ U | (\Pi, U) \in E^T \}.
$$

Note that (4) implies that $\Pi_f \leq \Pi_{\text{max}}$.

**Lemma 3** For any $(U, \Pi) \in E^T$,

1. $U + \Pi = \Pi_{\text{max}}$ if and only if $\Pi \in [\Pi_f, \Pi_{\text{max}}]$ and $U = \bar{U}^T(\Pi)$;

2. If $U + \Pi < \Pi_{\text{max}}$, then $K^T(U, \Pi) + U + \Pi < py - c$.

**Proof of Lemma 3**

Suppose that $U + \Pi = \Pi_{\text{max}}$. Then $\Pi_{\text{max}}$ is the maximum feasible total surplus, so $K^T(U, \Pi) = 0$ and hence $U = \bar{U}^T(\Pi)$. Define $\Pi'_f$ as the manager’s smallest equilibrium payoff such that $\bar{U}^T(\Pi'_f) + \Pi'_f = \Pi_{\text{max}}$, and denote $\Pi_H$ and $\Pi_L$ as the associated continuation profits. Note that $a = y$ for $(\bar{U}^T(\Pi'_f), \Pi'_f)$, and so

$$
\Pi'_f = p((1-\delta)(y - b) + \delta\Pi_H) + (1-p)\delta\Pi_L \\
\geq p(1-\delta)y + (1-p)\delta\Pi'_f,
$$

where the equality holds by (PK-A) and the inequality follows because (DE) implies $\delta\Pi_H \geq (1-\delta)b$, and $\Pi_L \geq \Pi'_f$ in order for sum of the manager’s and worker’s payoffs to equal $\Pi_{\text{max}}$. Rearranging this expression yields $\Pi'_f \geq \Pi_f$. 37
Now, suppose $\Pi \geq \Pi_f$, and consider the set of stationary strategies such that $a = 1$, $r = 0$ and $b \in \left[ \frac{c}{p}, \frac{\delta \Pi_f}{1-\delta} \right]$. It is straightforward to argue that all of these payments can be sustained in a relational contract. With $b = \frac{c}{p}$, the manager earns $\Pi_{\text{max}}$; with $b = \frac{\delta \Pi_f}{1-\delta}$, the manager’s payoff is $\Pi_f$. Therefore, $\hat{\Pi}^T(\Pi) + \Pi = py - c$ for any $\Pi \geq \Pi_f$. Combined with the result that $\hat{\Pi}'_f \geq \Pi_f$, we conclude that $\hat{\Pi}'_f = \Pi_f$ and that for any $\Pi \in [\Pi_f, \Pi_{\text{max}}]$, $\hat{\Pi}^T(\Pi) + \Pi = py - c$, which proves part 1.

Next, define

$$z \equiv \min \left\{ U + \Pi | K^T(U, \Pi) + U + \Pi = \Pi_{\text{max}} \right\}.$$ 

Suppose $z < \Pi_{\text{max}}$, and choose $(U, \Pi) \in E^T$ such that $U + \Pi = z$ and $K^T(U, \Pi) + U + \Pi = py - c$. Then it must be that $a = 1$ with probability 1, and moreover $U_L + \Pi_L + K^T(U_L, \Pi_L) = py - c$. Then summing (PK-A) and (PK-P) implies that

$$z = (1 - \delta)(py - c - pr) + \delta p (U_H + \Pi_H - U_L - \Pi_L) + \delta (U_L + \Pi_L)$$
$$\geq (1 - \delta)(py - c) + \delta (U_L + \Pi_L),$$

where the inequality follows from (TT). Since $z < py - c$, $U_L + \Pi_L < z$. But $U_L + \Pi_L + K^T(U_L + \Pi_L) = py - c$, yielding a contradiction. ■

**Lemma 4** The following hold:

1. For $\Pi \leq \Pi_f$,

$$\hat{\Pi}^T(\Pi) = \frac{\hat{\Pi}^T(\Pi_f)}{\Pi_f} \Pi.$$

2. For any $\Pi \in [0, \Pi_{\text{max}}]$, $K^T\left(\hat{\Pi}^T(\Pi), \Pi\right) = 0$.

3. For any $(U, \Pi)$ with $U + \Pi < py - c$, $K^T(U, \Pi) + \Pi$ is strictly increasing in $\Pi$. For $\Pi \geq \Pi_f$, $K^T(U, \Pi) + \Pi$ is strictly increasing in $U$.

**Proof of Lemma 4**

**Part 1:** Define $\xi = \max \left\{ \frac{U}{\Pi} | (U, \Pi) \in E^T \right\}$ and let $(U, \Pi)$ be such that $\frac{U}{\Pi} = \xi$. Lemma 2 implies that we can take $(U, \Pi) \in E_1$, and it is immediate from the definition of $\hat{\Pi}^T(\cdot)$ that we can take $U = \hat{\Pi}^T(\Pi)$. We claim that $\Pi = \Pi_f$. For any $\Pi > \Pi_f$, $\hat{\Pi}^T(\Pi) + \Pi = \Pi_{\text{max}}$, and so $\hat{\Pi}^T(\Pi)$ is strictly decreasing in $\Pi$ on this range. So $\frac{\hat{\Pi}^T(\Pi_f)}{\Pi_f} > \frac{\hat{\Pi}^T(\Pi)}{\Pi}$ for any $\Pi > \Pi_f$. 

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Consider $\Pi < \Pi_f$. To show this, we note two properties. First, $r = 0$ in any equilibrium with payoffs $(\tilde{U}^T(\Pi), \Pi)$, since otherwise we could decrease $r$ and increase $b$ so that $r + b$ is constant and the worker earns a strictly higher payoff than $\tilde{U}^T(\Pi)$.

Second, we claim that there exists an equilibrium giving $(\tilde{U}^T(\Pi), \Pi)$ in which $\Pi_H > \Pi_f$. Recall that $(\tilde{U}^T(\Pi), \Pi) \in E_1$, and note that either (DE) or the upper bound of (LL) must bind, since otherwise we could increase $b$ and hence increase the worker’s payoff. Suppose the upper bound of (LL) is slack, so $b < y$. From the public randomization device, $\tilde{U}^T(\Pi)$ is concave, so $\Pi + \tilde{U}^T(\Pi)$ is increasing in $\Pi$ (because it is constant for $\Pi \geq \Pi_f$). Therefore, consider increasing $b$ and $\frac{\delta}{1-\delta} \Pi_H$ by the same amount. Doing so holds the manager’s payoff constant and gives the worker a higher payoff. So there exists an equilibrium giving $(\tilde{U}^T(\Pi), \Pi)$ in which $b = y$. But then

$$\delta \Pi_H \geq (1-\delta)y > \frac{\delta p(1-\delta)y}{1-(1-p)\delta} = \delta \Pi_f,$$

as desired.

Given these two properties,

$$1 + \xi = \frac{\frac{U+\Pi}{\Pi}}{(1-\delta)(py-c) + \delta(py-c) + \delta(1-p)(U_L + \Pi_L)} \leq \frac{(1-\delta+\delta p)(py-c) + (1-p)\Pi_L(1+\xi)}{(1-\delta+\delta p)(py-c) + (1-p)\Pi_L(1+\xi)} \leq \frac{\tilde{U}^T(\Pi_f) + \Pi_f}{\Pi_f} \leq \frac{(1-\delta+\delta p)(py-c)}{(1-\delta)(py-c) + \delta(1-p)\Pi_L}$$

Here, the first equality follows from $\Pi_H > \Pi_f$ and so $\tilde{U}^T(\Pi_H) + \Pi_H = py - c$, the first inequality holds because $\frac{U_L + \Pi_L}{\Pi_L} \leq 1 + \xi$ by definition of $\xi$, the second inequality follows because (DE) implies that $\Pi \geq (1-\delta)py + \delta(1-p)\Pi_L$, the third inequality holds because $\delta(1-p)\Pi_L \geq 0$, and the final equality holds by definition of $\Pi_f$ and because $\tilde{U}^T(\Pi_f) + \Pi_f = py - c$.

We conclude that $\frac{\tilde{U}^T(\Pi_f)}{\Pi_f} = \xi$, as desired, which implies part 1 of Lemma 4 because $\tilde{U}^T(\Pi)$ is concave and so $\frac{\tilde{U}^T(\Pi)}{\Pi}$ is decreasing in $\Pi$, and strictly so unless $\tilde{U}^T(\cdot)$ is linear.
Part 2: Note that for $\Pi \geq \Pi_f$, Lemma 3 implies that $K^T(\tilde{U}^T(\Pi), \Pi) = 0$. For $\Pi < \Pi_f$, (8) holds with equality only if $\frac{\tilde{U}^T(\Pi)}{\Pi} = \xi$. But then $\Pi_L \geq \Pi_f$, implying that $K^T(U_L, \Pi_L) = 0$, and similarly $\Pi_H \geq \Pi_f$ so $K^T(U_H, \Pi_H) = 0$. Since $r = 0$ as well, $K^T(\tilde{U}^T(\Pi), \Pi) = 0$ in this range too.

Part 3: Lemma 3 and the concavity of $K^T(\cdot)$ imply that $K^T(U, \Pi) + U$ is strictly increasing in $U$ for $\Pi \geq \Pi_f$. Similarly, Lemma 3, concavity of $K^T$, and the fact that $\tilde{U}^T(\Pi)$ is maximized at $\Pi_f$ imply that $K^T(U, \Pi) + \Pi$ is strictly increasing whenever $U + \Pi < py - c$. ■

Given this characterization, we are prepared to prove our main result.

Proof of Proposition 4

Part 1: Suppose $U_H + \Pi_H < py - c$. First, we argue that $r + b = y$ whenever $U_H + \Pi_H < py - c$. Note that $\Pi_H < \Pi_f$. Suppose $r + b < y$, and consider an alternative that increases $r$ by $\epsilon > 0$ and increases $\Pi_H$ by $\frac{1-\delta}{\delta} \epsilon$. For $\epsilon > 0$ sufficiently small, this perturbation is feasible—in particular, Lemma 4 implies $(U_H, \Pi_H + \frac{1-\delta}{\delta} \epsilon) \in E^T$ because $\Pi_H < \Pi_f$—and it continues to satisfy the constraints of (P). Moreover,

$$\delta K^T \left( U_H, \Pi_H + \frac{1-\delta}{\delta} \epsilon \right) + (1-\delta) \epsilon > \delta K^T(U_H, \Pi_H)$$

by part 3 of Lemma 4. So the original equilibrium cannot be on the frontier $K^T(\cdot)$; contradiction.

Next, we show that $b = 0$. Suppose $b > 0$, and consider increasing $r$ and decreasing $b$ by $\epsilon > 0$, and increasing $U_H$ by $\frac{1-\delta}{\delta} \epsilon$. As before, this perturbation satisfies the constraints of (P). It is also feasible for sufficiently small $\epsilon > 0$, since $\Pi_H > \Pi_f$ from the proof of Lemma 4 and so $U_H < py - c - \Pi_H \leq \tilde{U}^T(\Pi_H)$.

Now, since $\Pi_H > \Pi_f$,

$$\delta \epsilon + \delta K \left( U_H + \frac{1-\delta}{\delta} \epsilon, \Pi_H \right) > \delta K(U_H, \Pi_H)$$

by part 3 of Lemma 4. So the creditor earns a strictly higher payoff in the perturbed equilibrium. Hence, $b = 0$ and so $r = y$. 

40
Part 2: Fix $S$, and consider the intersection of the line defined by varying $\Pi$ in $(S - \Pi, \Pi)$ and the set $E^T$. This intersection defines a line segment; the endpoint of that segment that maximizes $S - \Pi$ is defined by $\bar{U}^T(\Pi) = S - \Pi$, while the endpoint that maximizes $\Pi$ is defined by $S = \Pi$.

By concavity of $K^T(\cdot)$, part 2 of Proposition 4 is equivalent to the statement that, for any fixed and feasible $S$, $K^T(S - \Pi, \Pi)$ is weakly increasing in $\Pi$. By Lemma 2, it suffices to show this property for $(U, \Pi) \in E_1$, which by concavity of $K^T(\cdot)$ reduces to showing that $K^T(S - \Pi, \Pi)$ being maximized at $\Pi = S$.

For each $S$, let $\Pi^*(S)$ be the largest maximizer of $K^T(S - \Pi, \Pi)$. Towards contradiction, suppose $\Pi^*(S) < S$ for some $S$, and let

$$S \equiv \max_{\{S|\Pi^*(S)<S\}}\{S - \Pi^*(S)\}$$

as the $S$ that minimizes $\Pi^*(S)$ within this set. Consider the equilibrium that induces payoffs $(\bar{S} - \Pi^*, \Pi^*) \in E^T$. If $b > 0$ in this equilibrium, then we can decrease $b$ without violating the constraints of (P) because $U \equiv \bar{S} - \Pi^*(\bar{S}) > 0$, which would contradict that $\Pi^*(\bar{S})$ is the largest maximizer of $K^T(\bar{S} - \Pi, \Pi)$. So $b = 0$, which implies that

$$\bar{S} - \Pi^*(\bar{S}) = (1 - \delta)(-c) + \delta(pU_H + (1 - p)U_L)$$

by (PK-A).

Since $K^T(\cdot)$ is concave, for any $S$, $K^T(S - \Pi, \Pi)$ is (weakly) increasing in $\Pi$ for $\Pi < \Pi^*(S)$. Define $S_L \equiv \Pi_L + U_L$ and $S_H \equiv \Pi_H + U_H$. We claim that $\Pi_L \geq \Pi^*(S_L)$. Suppose $\Pi_L < \Pi^*(S_L)$, so that $U_L > 0$. Perturb $(U_L, \Pi_L)$ by decreasing $U_L$ and increasing $\Pi_L$ so that $S_L$ is constant. This alternative continues to satisfy the constraints in (P), weakly increases the creditor’s payoff because $K^T(S_L - \Pi_L, \Pi_L)$ is increasing in $\Pi_L$, and strictly increases the manager’s and decreases the worker’s payoffs. But then $\Pi^*(\bar{S})$ cannot be the largest maximizer of $K^T(\bar{S} - \Pi, \Pi)$. By a very similar argument, $\Pi_H \geq \Pi^*(S_H)$.

Consequently,

$$S - \Pi^*(S) \leq (1 - \delta)(-c) + \delta(p(S_H - \Pi^*(S_H)) + (1 - p)(S_L - \Pi^*(S_L)))$$

$$\leq (1 - \delta)(-c) + \delta(S - \Pi^*(\bar{S})).$$
where the first inequality follows from $U_H = S_H - \Pi_H \leq S_H - \Pi^*(S_H)$ and $U_L = S_L - \Pi_L \leq S_L - \Pi^*(S_L)$, and the second holds because $S - \Pi^*(S) \leq \tilde{S} - \Pi^*(\tilde{S})$ for any $S$ by definition of $\tilde{S}$. But then $\tilde{S} - \Pi^*(\tilde{S}) \leq -c$, which contradicts that the worker earns non-negative payoff. So $\Pi^*(S) = S$ for all $S$, proving the claim.

**Part 3:** By Lemma 2, it suffices to show that when $K(U, \Pi) > 0$, continuation play at some successor history lies in $E_0$ with positive probability. Suppose not; then $K(U, \Pi) + U + \Pi = py - c$. But then Lemma 3 (part 1) and Lemma 4 (part 2) imply that $K(U, \Pi) = 0$; contradiction. ■
B Details of Empirical Analysis

B.1 Sample Construction

We access the Amadeus database using WRDS. Since the required variables are often unavailable for small firms, we include only large and very large companies. A firm is considered large or very large if it meets at least one of the following conditions: more than 150 employees, operating revenue higher than 10 mln EUR or total assets of 20 mln EUR or more. We use the entire available time period, 1985-2017, but since Amadeus provides no more than 10 years of data for each firm, for most of firms our data comes from from 2006/07-2016/17. We keep the consolidated data whenever both consolidated and unconsolidated data are available. If a firm has more than one observation per year, we use the latest.

Capital is defined as the log of fixed assets and labor is defined as the log of cost of employees. We use operating revenue as a proxy for output, since sales entails more missing data. Our proxy for intermediate inputs is the log of material costs, and Industry is given by 2-digits codes from the NACE2 classification. We keep only manufacturing firms, which have NACE2 codes between 10 and 32, and we drop firms that have fewer than 5 yearly observations of capital. To reliably estimate TFP by (industry X country), we keep only those industry-country pairs that have at least 1000 firm-year observations with non-missing capital, labor, output and intermediate inputs measures. Our results with OLS-based TFP-R are similar if we use a threshold of 100 observations instead; our other measures, however, are more data-intensive and so frequently fail to converge if the number of observations is too small.

This restriction greatly reduces our working sample for two reasons. First, not all information is available for all countries. For example, firms in the United Kingdom do not report intermediate outputs and so are dropped. Second, smaller countries or industries might not have 1000 observations. Nevertheless, all but one industry (tobacco manufacturing) is included for at least one country, and all large European countries, except for the United Kingdom, are included in the final sample. Our final data has 127,703 observations. To make sure our results are not driven by outliers, we trim both regressands and regressors at the 5th and 95th percentile. Table 4 presents summary statistics for our final sample.
Table 4: Summary Statistics for Estimation Sample

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Std Dev</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leverage</td>
<td>0.140</td>
<td>0.114</td>
<td>0.112</td>
<td>0</td>
<td>0.553</td>
</tr>
<tr>
<td>ΔLeverage</td>
<td>-0.0024</td>
<td>-0.0024</td>
<td>0.0416</td>
<td>-0.1314</td>
<td>0.1472</td>
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<tr>
<td>Operating revenue (mln Euro)</td>
<td>60.0</td>
<td>25.3</td>
<td>125.7</td>
<td>0.5</td>
<td>8277.8</td>
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<tr>
<td>Fixed assets (Mln Euro)</td>
<td>16.1</td>
<td>6.4</td>
<td>27.5</td>
<td>0.3</td>
<td>223.5</td>
</tr>
<tr>
<td>Costs of employees (Mln Euro)</td>
<td>9.7</td>
<td>4.1</td>
<td>26.6</td>
<td>0.0</td>
<td>4115.3</td>
</tr>
<tr>
<td>Number of employees</td>
<td>140.18</td>
<td>90</td>
<td>135.36</td>
<td>15</td>
<td>775</td>
</tr>
<tr>
<td>Material costs (Mln Euro)</td>
<td>33.3</td>
<td>12.7</td>
<td>76.5</td>
<td>0.0</td>
<td>2733.5</td>
</tr>
<tr>
<td>Total assets (Mln Euro)</td>
<td>45.7</td>
<td>21.7</td>
<td>78.2</td>
<td>0.4</td>
<td>2756.6</td>
</tr>
<tr>
<td>TFP-R (OLS)</td>
<td>-0.059</td>
<td>-0.096</td>
<td>0.420</td>
<td>-0.903</td>
<td>1.099</td>
</tr>
<tr>
<td>ΔTFP-R (OLS)</td>
<td>-0.004</td>
<td>-0.005</td>
<td>0.147</td>
<td>-1.477</td>
<td>1.771</td>
</tr>
<tr>
<td>TFP-R (LP)</td>
<td>2.792</td>
<td>2.343</td>
<td>3.448</td>
<td>-4.109</td>
<td>11.681</td>
</tr>
<tr>
<td>ΔTFP-R (LP)</td>
<td>0.004</td>
<td>0.003</td>
<td>0.111</td>
<td>-3.485</td>
<td>3.609</td>
</tr>
<tr>
<td>TFP-R (GNR)</td>
<td>23.348</td>
<td>23.343</td>
<td>0.676</td>
<td>15.824</td>
<td>26.043</td>
</tr>
<tr>
<td>ΔTFP-R (GNR)</td>
<td>0.021</td>
<td>0.017</td>
<td>0.189</td>
<td>-5.363</td>
<td>5.524</td>
</tr>
</tbody>
</table>

Observations: 127703

B.2 TFP Calculations

We use three methods to calculate revenue-based TFP. Our first approach runs a linear regression of log output on log capital and log labor. We run this regression separately for each country-industry pair in our final sample, compute coefficients of capital and labor, and save the residuals as TFP measure. This method of estimating TFP is potentially biased, since firms might adjust their inputs in anticipation of productivity shocks. Two well-known proxy methods address this problem: Olley and Pakes (1996) and Levinsohn and Petrin (2003), which respectively use investment or intermediate inputs to identify the coefficient on labor, and then back out TFP using this estimate. Since Amadeus does not consistently report investment, we use Levinsohn and Petrin (2003) (henceforth LP) and proxy intermediate inputs using material costs. These calculations are performed using the `levpet` function in Stata. As a further test, we adopt the methods of Gandhi et al. (2017) (henceforth GNR), which address some of the weaknesses in these TFP measures.\(^{10}\) We gratefully acknowledge help from the authors of that paper, who provided the Stata code for our estimation.

\(^{10}\) We use this method rather than the alternative proposed by Ackerberg et al. (2015) because it is better suited to TFP calculations for a gross output production function.
### B.3 Alternative Weighting

Table 5: Changes in TFP and Changes in Leverage - Alternative Weightings

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆Leverage</td>
<td>-0.385</td>
<td>-0.301</td>
<td>-0.159</td>
<td>-0.0341</td>
<td>-0.416</td>
<td>-0.409</td>
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<tr>
<td></td>
<td>(0.0528)</td>
<td>(0.0590)</td>
<td>(0.0322)</td>
<td>(0.0664)</td>
<td>(0.0821)</td>
<td>(0.0988)</td>
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<tr>
<td>∆Leverage</td>
<td>-0.0320</td>
<td>0.00546</td>
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</tr>
<tr>
<td>(t-1)</td>
<td>(0.0276)</td>
<td>(0.0314)</td>
<td>(0.0207)</td>
<td>(0.0377)</td>
<td>(0.0249)</td>
<td>(0.0227)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>∆TFP-OLS</th>
<th>∆TFP-OLS</th>
<th>∆TFP-LP</th>
<th>∆TFP-LP</th>
<th>∆TFP-GNR</th>
<th>∆TFP-GNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆Leverage</td>
<td>-0.352</td>
<td>-0.284</td>
<td>-0.161</td>
<td>-0.0564</td>
<td>-0.368</td>
<td>-0.380</td>
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<tr>
<td></td>
<td>(0.0557)</td>
<td>(0.0577)</td>
<td>(0.0394)</td>
<td>(0.0504)</td>
<td>(0.0652)</td>
<td>(0.0872)</td>
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<tr>
<td>∆Leverage</td>
<td>-0.0378</td>
<td>-0.0105</td>
<td>-0.0256</td>
<td>-0.00187</td>
<td>-0.0132</td>
<td>-0.0299</td>
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<tr>
<td>(t-1)</td>
<td>(0.0304)</td>
<td>(0.0244)</td>
<td>(0.0176)</td>
<td>(0.0277)</td>
<td>(0.0260)</td>
<td>(0.0232)</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>∆TFP-OLS</th>
<th>∆TFP-OLS</th>
<th>∆TFP-LP</th>
<th>∆TFP-LP</th>
<th>∆TFP-GNR</th>
<th>∆TFP-GNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆Leverage</td>
<td>-0.387</td>
<td>-0.367</td>
<td>-0.147</td>
<td>-0.130</td>
<td>-0.442</td>
<td>-0.378</td>
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<tr>
<td></td>
<td>(0.0395)</td>
<td>(0.0327)</td>
<td>(0.0178)</td>
<td>(0.0171)</td>
<td>(0.0774)</td>
<td>(0.0632)</td>
</tr>
<tr>
<td>∆Leverage</td>
<td>-0.0606</td>
<td>-0.0425</td>
<td>-0.0242</td>
<td>-0.0227</td>
<td>-0.0515</td>
<td>-0.0376</td>
</tr>
<tr>
<td>(t-1)</td>
<td>(0.0255)</td>
<td>(0.0178)</td>
<td>(0.0270)</td>
<td>(0.0256)</td>
<td>(0.0232)</td>
<td>(0.0154)</td>
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| N        | 127703 | 127703 | 127703 | 127703 | 127703 | 127703 |
| Firm FE  | Yes    | -      | Yes    | -      | Yes    | -      |
| Year FE  | Yes    | -      | Yes    | -      | Yes    | -      |
| Industry×Year FE | Yes | - | Yes | - | Yes | - |

Top panel weights observations by total assets; middle panel weights by operating revenue and the bottom panel weights by number of employees. All columns use an unbalanced panel with firm fixed effects and binary variables for each year. Additional controls include total fixed assets and number of employees. Dependent variable is change in TFP-R which is calculated based on OLS specification (columns 1 and 3), Levinsohn and Petrin (2003) method (LP, columns 2 and 4) and Gandhi et al. (2017) method (GNR, columns 3 and 6). Standard errors are clustered on the industry level and reported in parentheses.
B.4 Including Lagged Productivity

Table 6: Changes in TFP and Changes in Leverage - Including Lag of Productivity

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>∆TFP-OLS</td>
<td>∆TFP-OLS</td>
<td>∆TFP-LP</td>
<td>∆TFP-LP</td>
<td>∆TFP-GNR</td>
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</tr>
<tr>
<td>∆Leverage</td>
<td>-0.427</td>
<td>-0.415</td>
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<td>-0.152</td>
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<td>(0.0470)</td>
<td>(0.0416)</td>
<td>(0.0225)</td>
<td>(0.0194)</td>
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<td>(0.0644)</td>
</tr>
<tr>
<td>∆Leverage (t-1)</td>
<td>-0.198</td>
<td>-0.144</td>
<td>-0.0891</td>
<td>-0.0723</td>
<td>-0.205</td>
<td>-0.123</td>
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<tr>
<td></td>
<td>(0.0322)</td>
<td>(0.0240)</td>
<td>(0.0194)</td>
<td>(0.0195)</td>
<td>(0.0434)</td>
<td>(0.0271)</td>
</tr>
<tr>
<td>∆Dep. Var. (t-1)</td>
<td>-0.247</td>
<td>-0.155</td>
<td>-0.291</td>
<td>-0.186</td>
<td>-0.267</td>
<td>-0.161</td>
</tr>
<tr>
<td></td>
<td>(0.0136)</td>
<td>(0.0183)</td>
<td>(0.0121)</td>
<td>(0.0254)</td>
<td>(0.0235)</td>
<td>(0.0288)</td>
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<tr>
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<td>123818</td>
<td>123818</td>
<td>123818</td>
<td>123811</td>
<td>123811</td>
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<tr>
<td>SD of Dep Var</td>
<td>0.12</td>
<td>0.12</td>
<td>0.08</td>
<td>0.08</td>
<td>0.16</td>
<td>0.16</td>
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<tr>
<td>SD of ∆Lev</td>
<td>0.044</td>
<td>0.044</td>
<td>0.044</td>
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<td>Industry × Year</td>
<td>-</td>
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<td>-</td>
<td>Yes</td>
<td>-</td>
<td>Yes</td>
</tr>
</tbody>
</table>

All columns use an unbalanced panel with firm fixed effects and binary variables for each year. Additional controls include total fixed assets and number of employees. Dependent variable is change in TFP-R which is calculated based on OLS specification (columns 1 and 3), Levinsohn and Petrin (2003) method (LP, columns 2 and 4) and Gandhi et al. (2017) method (GNR, columns 3 and 6). In the bottom rows, standard deviation of dependent variable and of change in leverage is a within-firm value and median across all firms is presented. Standard errors are clustered on the industry level and shown in the parentheses.

B.5 Magnitudes Calculations

The most conservative estimate of the effect of leverage comes from the LP measure of TFP. The standard deviation of log-change in leverage in our final sample is 0.044, while the within-firm standard deviation of TFP changes for the median firm equals 0.081 (with a mean of 0.133). Using our estimated coefficient on leverage, we conclude that a one standard deviation increase in leverage is correlated with a decrease in TFP of 0.007, which equals 8.7% of the median standard deviation.
The other specifications yield larger estimates for this correlation. For example, the GNR measure of TFP has coefficient of -0.475, with a median within-firm standard deviation in TFP of 0.156. The implied magnitude of the effect of a one standard deviation increase in leverage is therefore 13.3% of the median standard deviation in TFP. For our OLS measure of TFP, the magnitude is 15.3%. From Table 1, the effects of the previous year’s leverage on productivity are about one quarter the size of these estimates.
C Internet Appendix: Collusion and Truth-telling

This section defines an equilibrium refinement that captures collusion between the manager and worker, then shows that Definition 1 is sufficient to incorporate this notion of collusion. This sufficiency argument relies on binary effort.

Definition 2 A SPE $\sigma^*$ is a collusion equilibrium if, after the creditor accepts the equilibrium formal contract $(R, l(\cdot))$, continuation play maximizes the manager’s payoff among all continuation SPE.

The intuition for a collusion equilibrium follows from the following heuristic timing. Suppose that the manager first negotiates with the creditor to secure a loan. After the creditor signs this formal contract, however, the manager can sit down with the worker and propose a continuation equilibrium. During her negotiations with the creditor, the manager cannot credibly promise to choose an equilibrium in which the worker punishes her for failing to repay the creditor. Therefore, Definition 2 captures the idea that the creditor does not have a seat at the table when the manager and worker decide on a relational contract. Essentially, such equilibria resemble “multi-tier” contracting problems (Tirole (1986); DeMarzo et al. (2005)), with the important differences that the game is infinite-horizon and contracts must be self-enforcing.

One immediate implication of this definition is that liquidation must occur with positive probability whenever the project is funded in a collusion equilibrium. If it did not, then the manager and worker could agree to never repay the creditor for her initial loan. We prove a stronger result: there exists a profit-maximizing truth-telling equilibrium that is also a collusion equilibrium.

Proposition 5 Consider the model with binary effort from Section 4. There exists a profit-maximizing truth-telling equilibrium that is a collusion equilibrium.

Proof: Proposition 4 says that there exists a truth-telling equilibrium in which $a_t = y$ in every period until the project is liquidated. Consider this equilibrium, and note that it maximizes total surplus given the formal contract $l(\cdot)$. It suffices to show that, immediately after the creditor agrees to $l(\cdot)$, no alternative equilibrium gives the manager a strictly higher expected continuation payoff.
Consider the following “ancillary game,” which has two players: a firm and a creditor. In each period, the firm chooses \( a_t, r_t, \) and \( b_t \), bears the cost \( c a_t \), and earns the sum of the manager and worker’s utility. The creditor’s actions and payoffs are unchanged. Fix \( l(\cdot) \) as in the original game. Since the firm earns \( \Pi + U \) in each period, (3) implies that it has no one-shot deviation in \( r_t \). It has no one-shot deviation in \( a_t \) either, since regardless of \( l(\cdot) \), \( a_t = 1 \) and \( r_t = 0 \) generates a strictly higher sum of manager and worker utilities than \( a_t = r_t = 0 \). Finally, it has no deviation in \( b_t \), which does not affect the sum of the manager’s and worker’s payoff. The one-shot deviation principle applies to this ancillary game, so the firm’s payoff is maximized by choosing \( a_t, r_t, \) and \( b_t \) as specified in the collusive-proof equilibrium.

Now, return to the three-player game. The preceding argument implies that after the creditor agrees to \( l(\cdot) \), \( \Pi + U \) is maximized by following the equilibrium strategy. But \( U = 0 \) immediately after the creditor agrees to \( l(\cdot) \). Since \( U \geq 0 \), \( \Pi \) is bounded above by \( \Pi + U \), and moreover we have argued that \( \Pi = \Pi + U \) at the point where \( \Pi + U \) is maximized. Therefore, \( \Pi \) is maximized by following the equilibrium, and so there exists no alternative equilibrium that generates strictly higher profit given \( l(\cdot) \). The manager is willing to choose \( l(\cdot) \) in equilibrium if all other contracts lead to the worker choosing \( a_t = 0 \) in every \( t \). So this profit-maximizing truth-telling equilibrium is a collusion equilibrium, as desired.

Proposition 5 exhibits a profit-maximizing truth-telling equilibrium that also satisfies Definition 2. However, this result does not say that that equilibrium is a profit-maximizing collusion equilibrium. It is in that sense that Definition 1 is a sufficient but not a necessary condition for a collusion equilibrium—the manager can certainly do at least as well in a collusion equilibrium, but it is an open question whether she could do strictly better.
D  Internet Appendix: Truth-Telling with Continuous Effort

This section considers truth-telling equilibria in the game with continuous effort.

In the model from Section 2 with \( a_t \in \mathbb{R}_+ \), let \( K^T(U, \Pi) \) be the truth-telling equilibrium payoff frontier for the creditor, given worker and manager payoffs \( U \) and \( \Pi \), respectively. Then \( K^T(\cdot) \) solves problem \((P^\prime)\), defined as maximizing \((1)\) subject to \((PK-A)-(LL), (TT)\), and

\[
(U, \Pi) \in E^T,
\]

where \( E^T \) is defined analogously to Section 4.

Define \( \bar{U}^T(\cdot) \) analogously to the proof of Proposition 4, let \( \Pi_{max} \equiv pa_{max} - c(a_{max}) \), and define

\[
\Pi_f \equiv \frac{p(1-\delta)a_{max}}{1-(1-p)\delta}.
\]

We prove the following result.

**Proposition 6** There exists a non-empty, open set \( B \subseteq E^T \) such that \( K^T(U, \Pi) = K(U, \Pi) \) if and only if \((U, \Pi) \in B\), and otherwise \( K^T(U, \Pi) < K(U, \Pi) \). Moreover,

1. **Frontload creditor payments when truth-telling changes behavior:** Whenever \((U, \Pi) \) is such that \( \Pi_H < \Pi_f \), \( b + r = y \). If \((U_H, \Pi_H) \notin B\), then \( b = 0 \) and \( r = y \).

2. **Aggregation result fails:** There exists \( \bar{\Pi} : \mathbb{R}_+ \to \mathbb{R}_+ \) such that \( s + K(s - \Pi, \Pi) \) is strictly increasing in \( \Pi \) for \( \Pi < \bar{\Pi}(s) \) and is constant for \( \Pi \geq \bar{\Pi}(s) \).

The equilibrium payoff frontier can be split into two regions. If \((U, \Pi) \in B\), then it is as if \((TT)\) does not bind, in which case equilibrium payoffs are identical to payoffs without this constraint. If \((U, \Pi) \notin B\), then \((TT)\) constrains equilibrium play and leads to lower equilibrium payoffs. Intuitively, \((TT)\) binds when the creditor is owed money. Consequently, and as in Section 4, part 1 of Proposition 6 says that the manager strictly prioritizes repaying the creditor whenever continuation play lies in \( B \). Part 2 of this result suggests that productivity improves as the manager repays the creditor, so long as \( \Pi \) is not too large.
Proof of Proposition 6

We first state a result analogous to Lemma 2.

Lemma 5 Define

$$ E_1 \equiv \{(U, \Pi) \in E^T \exists \text{ a solution to } (P) \text{ with } a > 0 \} \subseteq E^T, $$

and let $E_0 \equiv E^T \setminus E_1$. Then $(0, 0) \in E_0$ and $K^T(0, 0) = 0$, so that $(0, 0)$ can be supported by liquidating the firm. Moreover, any $(U, \Pi) \in E_0$ can be implemented by randomizing between $(0, 0)$ and some $(U', \Pi') \in E_1$.

We omit the proof of this lemma, which follows very similar lines to the proof of Lemma 2.

Lemma 6 The following hold:

1. $K^T(U, \Pi) = 0$ whenever $U = \tilde{U}^T(\Pi)$.

2. For all $\Pi \in [0, \Pi_{max}]$, $\tilde{U}^T(\Pi) = \tilde{U}(\Pi)$.

Proof of Lemma 6

Part 1 follows the same argument as the proof of Lemma 1, since the perturbation used there decreases $r$ and so relaxes (TT).

For part 2, for any $(U, \Pi) \in E^T$ with $U = \tilde{U}^T(\Pi)$, we have $K(U, \Pi) = 0$ by part 1. Consequently, $\tilde{U}_L(\Pi_L) = U_L$ and $\tilde{U}_H(\Pi_H) = \Pi_H$, since otherwise $pK(U_L, \Pi_L) + (1-p)K(U_H, \Pi_H) > 0$. Then $r = 0$ in all subsequent periods. But in the relaxed problem with $r_t = 0$ and without (TT), yields $\tilde{U}^T(\Pi) = \tilde{U}(\Pi)$. In this relaxed problem, $U_H + \Pi_H \geq U_L + \Pi_L$ and so the solution to the relaxed problem satisfies (TT) as well. ■

Lemma 7 The following hold:

1. $K^T(U, \Pi) \leq K(U, \Pi)$ for all $(U, \Pi) \in E^T$.

2. $K^T(U, \Pi) < K(U, \Pi)$ for all $(U, \Pi) \in E^T$ such that $U < \tilde{U}^T(\Pi)$ and $\Pi \geq \Pi_f$.

3. For each $\Pi < \Pi_f$, there exists $g(\Pi) < \tilde{U}^T(\Pi)$ such that $K^T(U, \Pi) = K(U, \Pi)$ for all $(U, \Pi)$ satisfying $U \geq g(\Pi)$. 

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Proof of Lemma 7

**Part 1:** By Lemma 5, it suffices to show this for \((U, \Pi) \in E_1\). Note that \(K^T(\cdot)\) satisfies the Blackwell sufficient condition and so can be obtained through a sequence of approximations. Let \(K^T_0(U, \Pi) = 0\) for all \((U, \Pi) \in E_1\), and for all \(s > 0\), define

\[
K^T_s(U, \Pi) = \Gamma^T K^T_{s-1}(U, \Pi),
\]

where \(\Gamma^T\) is the operator induced by the problem \((P')\). Then \(K^T(U, \Pi) = \lim_{s \to \infty} K^T_s(U, \Pi)\).

Similarly, let \(K_0(U, \Pi) = 0\) for all \((U, \Pi) \in E\), and define

\[
K_s(U, \Pi) = \Gamma K(U, \Pi)
\]

where \(\Gamma\) is the operator induced by the SPE problem. Then for all \(s \geq 0\), \(K_s(U, \Pi) \geq K^T_s(U, \Pi)\) because \(\Gamma^T\) entails strictly more constraints than \(\Gamma\). Consequently, \(K(U, \Pi) \geq K^T(U, \Pi)\).

**Part 2:** Suppose \(U < \bar{U}(\Pi)\) and \(\Pi \geq \Pi_f\). In this case, \(K(U, \Pi) + U + \Pi = \Pi_{max}\) by the proof of Proposition 3. Therefore, if we define

\[
z \equiv \min \left\{ U + \Pi | K^T(U, \Pi) + U + \Pi = \Pi_{max} \right\},
\]

then it suffices to show that \(z = \Pi_{max}\).

Suppose to the contrary that \(z < \Pi_{max}\), and choose \((U, \Pi)\) such that \(U + \Pi = z\) and \(K^T(U, \Pi) + U + \Pi = \Pi_{max}\). Then it must be that \(K^T(U_L, \Pi_L) + U_L + \Pi_L = \Pi_{max}\). However, summing (PK-A) and (PK-P) and applying (TT) yields

\[
U + \Pi = (1 - \delta)(pa_{max} - c(a_{max}) - pr) + \delta p(U_H + \Pi_H - U_L - \Pi_L) + \delta(U_L + \Pi_L) \geq (1 - \delta)(pa_{max} - c(a_{max})) + \delta(U_L + \Pi_L).
\]

Since \(U + \Pi = z < \Pi_{max}\), \(U_L + \Pi_L < z\). But \(K^T(U_L, \Pi_L) + U_L + \Pi_L = \Pi_{max}\), which contradicts the definition of \(z\). But it is clear that \(z \leq \Pi_{max}\), so \(z = \Pi_{max}\).
Part 3: Note that $K^T(U, \Pi) \leq K(U, \Pi)$ and $\frac{\partial K}{\partial U} = -1$ by Lemma 1. Since $K^T(\cdot)$ is concave, it therefore suffices to show that, for each $\Pi < \Pi_f$, there exists some $g < \tilde{U}^T(\Pi)$ such that $K(g, \Pi) = K^T(g, \Pi)$.

Proposition 3 implies one solution to the problem without (TT) is

$$y = \left(\frac{1-\delta(1-p)}{1-\delta} \right) \frac{\Pi}{p},$$

$$r = \left(\frac{\delta(1-p)(\tilde{U}(\Pi) - g)}{1-\delta(1-p)} \right),$$

$$b = a - r$$

$(U_L, \Pi_L) = (U, \Pi)$, and $(U_H, \Pi_H) = \left(\tilde{U} \left(\frac{1-\delta}{\delta} a\right), \frac{1-\delta}{\delta} a\right)$.

Since $\Pi < \Pi_f$, this solution satisfies $\Pi_H + U_H - \Pi_L - U_L > 0$, independent of $g$. Therefore, (TT) is satisfied for $g$ sufficiently close to $\tilde{U}(\Pi)$, so this solution also solves (P'). Hence, $K^T(g, \Pi) = K(g, \Pi)$. ■

Now, we can extend the function $g(\cdot)$ by setting $g(\Pi) \equiv 0$ for $\Pi \geq \Pi_f$, so that $K^T(U, \Pi) = K(U, \Pi)$ if and only if $U \geq g(\Pi)$.

Lemma 8 For any $(U, \Pi) \in E_1$, $U' > U$, and $\Pi' > \Pi$,

1. If $(U', \Pi) \in E_1$, then $K^T(U', \Pi) + U' \geq K^T(U, \Pi) + U$, and strictly so if $U' < g(\Pi)$;

2. If $(U, \Pi') \in E_1$, then $K^T(U, \Pi') + \Pi' > K^T(U, \Pi) + \Pi$ unless $K^T(U, \Pi) = 0$.

Proof of Lemma 8

Since $K^T(\cdot)$ is concave, it suffices to establish these properties at $(U, \Pi)$ satisfying $U = \tilde{U}^T(\Pi)$.

Part 1: This result immediately follows from two facts: (i) $K(U, \Pi) + U$ is constant in $U$ by Lemma 1, and (ii) $K^T(U, \Pi) < K(U, \Pi)$ for all $(U, \Pi)$ such that $U \leq g(\Pi)$.

Part 2: For this property, it suffices to consider $\Pi \geq \arg \max_{\Pi} \tilde{U}^T(\Pi)$. Note that if $\Pi > \Pi_f$ and $U = \tilde{U}^T(\Pi)$, $K(U, \Pi) + \Pi$ is constant in $\Pi$. But $K^T(U, \Pi) < K(U, \Pi)$ whenever $K^T(U, \Pi) > 0$ in this range, so the result obtains.
For $\Pi \leq \Pi_f$, recall that $\bar{U}^T(\Pi) + \Pi$ is strictly increasing in $\Pi$. Therefore, holding $U$ fixed at $\bar{U}^T(\Pi)$ and applying Lemma 1 implies that $\Pi + K(\Pi, U)$ is strictly increasing in $\Pi$. Since $K^T(U, \Pi) \leq K(U, \Pi)$, we conclude that $K^T(U, \Pi) + \Pi$ is also strictly increasing in $\Pi$. ■

We are now prepared to prove the two parts of Proposition 6.

**Proof of Proposition 6, Part 1**

It suffices to consider $(U, \Pi) \in E_1$. Suppose $\Pi_H < \Pi_f$.

First, we consider the case with $U_H < \bar{U}^T(\Pi_H)$. Suppose to the contrary that $r + b < y$, and consider the perturbation $r' = r + \frac{\delta}{1-\delta}\epsilon$, $\Pi'_H = \Pi_H + \epsilon$, with all other variables remaining the same. This perturbation satisfies the constraints of $(P')$, and in particular is feasible for sufficiently small $\epsilon > 0$ because $U_H < \bar{U}^T(\Pi_H)$. But

$$\delta p\epsilon + \delta p \left( K^T(U_H, \Pi_H + \epsilon) - K^T(U_H, \Pi_H) \right) > 0$$

by part 2 of Lemma 8. Contradiction of $K^T(U, \Pi)$ maximizing the creditor’s payoff given $(U, \Pi)$.

Next, suppose $U_H = \bar{U}^T(\Pi_H)$. Then it must be that $\Pi_H > 0$, since otherwise $U_H = U_L = \Pi_L = 0$, so $a = 0$ and hence $(U, \Pi) \notin E_1$. If $b + r < y$, consider the alternative with $b' = b + \frac{\delta}{1-\delta}\epsilon$, $U'_H = U_H - \epsilon$, and $\Pi'_H = \Pi_H + \epsilon$. This change continues to satisfy the constraints of $(P')$. Moreover, it is feasible and strictly increases the creditor’s payoff because $\bar{U}^T(\Pi) + \Pi$ is strictly increasing in $\Pi$ for $\Pi < \Pi_f$. Contradiction of $K^T(U, \Pi)$ maximizing the creditor’s payoff.

Now, we have already argued that for any $(U, \Pi) \in \mathcal{B}$, $U < g(\Pi)$ and hence $\Pi < \Pi_f$. Therefore, $r + b = y$ by the previous argument. Now suppose that $b > 0$, and consider the perturbation $r' = r + \frac{\delta}{1-\delta}\epsilon$, $b' = b - \frac{\delta}{1-\delta}\epsilon$, and $U'_H = U_H + \epsilon$, with all other variables remaining the same.

For small enough $\epsilon > 0$, this perturbation is feasible and continues to satisfy the constraints of $(P')$. Moreover, the creditor’s payoff increases by

$$\delta p\epsilon + \delta p \left( K^T(U_H + \epsilon, \Pi_H) - K^T(U_H, \Pi_H) \right) > 0,$$

where the inequality holds by part 1 of Lemma 8, since $U_H < g(\Pi_H)$. ■
Proof of Proposition 6, Part 2

By Lemma 5, $K^T(U, \Pi) = 0$ when $U = \tilde{U}^T(\Pi)$. Hence, for all $s$, $K(s - \Pi, \Pi)$ is minimized at $\Pi$ such that $s = \tilde{U}^T(\Pi) + \Pi$.

Define
\[
k(s) = \max \{ K(U, \Pi) | U + \Pi = s \} \quad \text{and} \quad u(s) = \min \{ U | K(U, \Pi) = k(s) \text{ and } U + \Pi = s \}.
\]

Concavity of $K(\cdot)$ along the line segment $U + \Pi = s$ implies that it suffices to rule out $u(s) > 0$. Suppose to the contrary that $u(s) > 0$ for some $s$, and let $u^* = \max_s \{ u(s) \}$. Let the associated payoffs be $(u^*, \pi^*) \in E^T$ and the surplus level be $s^*$.

Given $u^* > 0$, $b > 0$ because otherwise we could decrease the payment to the worker and continue to satisfy the constraints of (P'), which would violate the definition of $u^*$. Given that $b = 0$, (PK-A) implies that
\[
u^* = (1 - \delta)(-c(a)) + \delta (pU_H + (1 - p)U_L).
\]

Define $s_L = U_L + \Pi_L$. We claim that $U_L \leq u(s_L)$. If instead $U_L > u(s_L)$, then we can perturb $(U_L, \Pi_L)$ to $(U_L - \epsilon, \Pi_L + \epsilon)$ to decrease the worker’s payoff while increasing the manager’s creditor’s payoff and continuing to satisfy the other constraints of (P’). This perturbation again violates the definition of $u^*$. By a very similar argument, we can show that $U_H \leq u(s_H)$ for $s_H = U_H + \Pi_H$.

Then
\[
u^* \leq (1 - \delta)(-c(a)) + \delta (pu(s_H) + (1 - p)u(s_L)) \leq (1 - \delta)(-c(a)) + \delta u^*,
\]

where the first inequality follows by the previous paragraph, and the second inequality follows because $u^* = \max_s \{ u(s) \}$. Therefore, $u^* \leq -c(a)$, which is a contradiction of (IC). So $u(s) = 0$ for all $s$. ■