We consider the choice between stocks and options to provide effort incentives to a risk-averse manager. We show that stocks can dominate options as a means of motivation only if nonviability risk is substantial, as in financially distressed firms or start-ups. Options dominate stocks for other firms. These results hold regardless of the existing portfolio of the manager. We provide empirical evidence that higher bankruptcy risk is indeed correlated with more use of stock.

Although stock-based compensation has steadily increased over the last decade, views clearly diverge on the choice between stocks and options. For example, in February 2004, IBM announced that instead of “at the money” stock options, it would issue options for which the exercise price exceeds the current market price by 10%. In contrast, Microsoft announced in July 2003 that it would no longer use stock options and would favor restricted stock instead, effectively lowering the strike price to zero. In March 2004, as one-time insurance giant Conseco emerged from the third largest bankruptcy filing in US history, it announced that its top executives made nearly $34 million in awards primarily from restricted stock granted while the company was under Chapter 11.

The goal of this article is to shed light on the choice between stocks and options in compensation design. To do this, we study a principal-agent relation between risk neutral investors and a risk-averse and effort-averse manager.

We depart from the standard model (Holmström, 1979) in three ways. First, for most of the article we assume that the manager is compensated using only salary, stock, and options. This seems realistic given the contracts one actually
Ross (2004) takes a similar approach in studying the risk attitude of managers compensated by options.

Second, we assume that the base salary must be non-negative, and that at the time the firm is making its choice of compensation, the manager may already have the rights to a nonrevokable bundle of previously granted stocks and options. The design problem facing the firm is how many stocks or options to add to this bundle. This *minimum payment constraint* again seems realistic: Real-world contracts rarely either specify that the manager give the firm money or involve revoking stocks or options previously granted.

It is critical to note that in this setting, the participation constraint may or may not bind (since, e.g., payments can never be negative, but may for incentive reasons be very substantial). It does seem that in many real-life cases, CEOs’ compensation inside the firm exceeds outside opportunities.¹ For our theoretical results, we focus on this case. Broadly similar results obtain when the participation constraint binds.

Third, we allow the stock price to be zero with a non-negligible probability.² We refer to this as “nonviability risk.” A central example is bankruptcy, but nonviability risk can also describe the demise of firms that are unable to market their products or raise additional capital to finance their operations or further product development. This is common for start-ups.

Our main theoretical result reveals a significant connection between the optimal choice between stocks and options and the role of managerial effort in maintaining the viability of the firm. If nonviability is not a major issue or is out of managerial control, then stock options always dominate stocks. In contrast, if a key effect of increased managerial effort is to lower the probability of firm demise, then stocks can dominate options. Thus, start-ups might find it optimal to use stock in early stages but migrate to option-based compensation post-IPO, since nonviability risk is then likely to be smaller. Similarly, foundering firms may wish to use restricted stock but move toward options as they emerge from bankruptcy.

The intuition for this result is simple. Note first that the contribution to the manager’s incentives of a small change in stock price from any given starting point depends both on how quickly her compensation changes and on her marginal utility of income given that stock price level. With fully general contracts (in particular, ones that can pay arbitrarily negative amounts), the main thing connecting compensation at one stock price to compensation at another is the participation constraint. One can always make the agent have strong incentives over a region by essentially “spinning” the contract through that region. When payments are bounded below, as with stocks or options, the story is different: The more that compensation is responsive to stock price changes at one stock price level, the higher is the manager’s wealth at all higher

¹ This can also be the result of managerial entrenchment as in Zwiebel (1996).
² As seems reasonable, we assume that the distribution over stock prices is atom-free except at zero.
stock price levels. Since higher wealth translates into a lower marginal utility of income, the manager is less motivated by compensation changes at those higher levels. There is thus a real trade-off between providing more motivation at one stock price and the ability to provide motivation at higher stock prices.

So, let us consider the effect of increasing the exercise price on an option (or of replacing stock with an option with a small positive strike price). Above the exercise price, the risk-averse manager is less wealthy and hence her utility is more responsive to stock price changes. But there is also a larger range over which the manager is numb to changes in the stock price. The key is that if nonviability is not a major issue, then increasing the numb region will have little negative effect on the manager’s incentives. If nonviability is important, then an exercise price of zero (i.e., stock) may be optimal, because it may be near zero that the price distribution is most affected by managerial effort.3

The trade-off between increasing incentives at one stock price level and making incentives less effective at higher stock price levels can be extreme. An ill-designed options contract can make a risk-averse manager of a healthy company “too rich too soon,” and hence very unresponsive to incentives. In particular, granting managers more stocks or options with a low exercise price can make them exert less effort.

Our key result is not an artifact of the simple contract space. We show that when contracts can be arbitrary (but are subject to a minimum payment constraint), new compensation should consist of base salary only for all price realizations below a critical level at which effort is most influential on the stock price. This critical level is significantly larger than zero for healthy firms, and can be zero only if nonviability risk is substantial.4

Our model predicts that higher bankruptcy risk should be associated with a higher tendency by firms to compensate their executives by stock instead of stock options. We test this prediction empirically. Using data from 1992 to 2004, we show a significant relation between several proxies for bankruptcy risk and both the likelihood that a firm uses restricted stock and the fraction of total-stock-based compensation made up of restricted stock. In particular, firms with lower Altman’s Z-Score, higher KMV-Merton expected default probabilities, and lower debt ratings are more likely to compensate their CEOs by restricted stock. Our empirical formulation controls for firm characteristics, prior stock ownership by the manager, and differences in both the taxation and the expensing rules of restricted stock compared to stock options.

Section 1 discusses the related theoretical literature. Section 2 presents the model. Section 3 studies the choice between stocks and options. Section 4 discusses general optimal contracts and introduces the notion of the critical

3 We work in a static setting. A working paper (Kadan and Swinkels, 2006) shows that similar results hold in a dynamic setting in which compensation can take place over multiple periods.

4 See Jewitt et al. (2007) for a discussion of the technical issues involved with characterizing a solution when contracts can be arbitrary.
price. Section 5 discusses related empirical papers and presents the empirical evidence. Section 6 concludes. The Appendix contains proofs omitted from the text.

1. Related Theoretical Literature

Theoretical research on option-based compensation has traditionally focused on the effect of options on managerial risk attitudes. The traditional view is that options induce more risk taking (Jensen and Meckling, 1976). More recently, Carpenter (2000) and Ross (2004) showed that the effect of option compensation on risk taking is ambiguous and depends on the manager’s utility function. While we look at effort rather than risk taking, our results are complementary to theirs both in that changes in compensation may affect the effort of a risk-averse manager in unexpected ways and in that higher effort (induced by changes in compensation) can lead to higher or lower volatility of the stock price.

Lambert et al. (1991) study the divergence between the value the market and an undiversified, risk-averse manager place on stock options. Hall and Murphy (2000, 2002) use this framework to derive optimal exercise prices for stock options, and to analyze the cost, value, and pay-performance sensitivity of nontradable options held by executives. These papers do not model effort choice and do not consider nonviability risk.

Hemmer et al. (2000) study restrictions on price distributions and utility functions that induce a convex optimal contract in a moral hazard problem without a minimum payment restriction and bankruptcy risk. Our results show that even when the optimal contract without a minimum payment constraint is concave, introducing this constraint may result in a convexity of the contract as long as nonviability risk is not a main issue.

While we are unaware of previous theoretical work linking nonviability risk to optimal compensation, the impact of capital structure on managerial compensation has been studied. John and John (1993) study the interaction between the agency relationship between shareholders and managers and the risk-shifting effect between shareholders and debt holders, showing that the optimal sensitivity of the manager’s pay to the stock price is decreasing in the debt level. Berkovitch et al. (2000) offer a model studying how risky debt affects both the probability of managerial replacement and the manager’s wage if he is retained by the firm. Cadenillas et al. (2004) study the incentive effects of debt on a risk-averse manager in a dynamic continuous time framework. Their numerical results suggest that options are optimal for managers that are more effective in affecting firm value through effort, and in firms with high momentum, large firms, and firms for which additional volatility implies a small decrease in returns.

Finally, Innes (1990) studies debt contracts in a moral hazard setting with a wealth-constrained agent. Innes does not allow for nonviability risk or for risk aversion by the agent. Each is at the heart of our results. Lambert and Larcker
(2004) discuss a principal-agent problem with limited liability, but without nonviability risk. In their framework, the optimal contract is always option-like. We show that stock options may or may not dominate stocks depending on nonviability risk.5

2. Model

We study a principal-agent model between investors (the principal) and a manager (the agent). Our focus is on incentives for effort rather than, say, incentives to choose among projects.

A set of risk neutral investors own a firm. The firm employs a manager. The manager is risk-averse over her final wealth $w$, with utility $u(w)$. We assume $u(w)$ is twice continuously differentiable, with $u' > 0$, $u'' < 0$.

The manager chooses effort level $e \in [0, \bar{e}]$. Effort is unobservable to the firm. Effort $e$ costs the manager $\psi(e)$. We assume $\psi(\cdot)$ is twice continuously differentiable, with $\psi' > 0$, $\psi'' > 0$. Utility is additively separable: The net utility of a manager who chooses effort level $e$ and has terminal wealth $w$ is $u(w) - \psi(e)$.

Once $e$ is chosen, there is a realization of the terminal value of the firm $x \in [0, \bar{x}]$ that is stochastically related to $e$. This value is then divided between the investors and the manager.6 We will typically refer to $x$ simply as the stock price. The distribution of $x$ given effort level $e$ by the manager is $F(x \mid e)$. We assume that $F(\cdot \mid \cdot)$ is twice continuously differentiable and has full support for each $e$. This rules out atoms in values except at zero, and implies that $F$ has a density $f$ on $(0, \bar{x}]$. Higher effort moves the distribution of stock prices to the right in the sense of strict first-order stochastic dominance (FOSD). That is, $F_e(x \mid e) < 0$ except when $x = \bar{x}$ and possibly when $x = 0$.

A key feature of our model is that we allow nonviability risk as, for example, bankruptcy risk or the failure of a start-up. We capture this by allowing a positive probability of a zero stock price, so that a firm with a risk of nonviability has $F(0 \mid e) > 0$. Note that $F_e(0 \mid e)$ is the marginal effect of effort on the probability that the firm becomes nonviable and that for $x > 0$, $F(x \mid e) = F(0|e) + \int_0^x f(y|e)dy$.

The manager is compensated as a function $\pi(x)$ of the stock price $x$. We assume $\pi(\cdot)$ to be continuous and piecewise differentiable. For given $e$ and $\pi$,

5 In Stoughton and Wong (2003), industry competition, accounting choice, and the possibility to reprice options play a role in optimal compensation. Kadan and Yang (2005) study the earnings management implications of executive stock options. Cadenillas et al. (2005) show that options can serve as a screening mechanism for executives.

6 Thinking of $x$ as a value to be divided between the investors and the manager avoids the technical complications of a setting where compensation is a function of stock price, but (even holding effort fixed) the distribution over stock price is a function of compensation. For practical purposes, the difference is typically minor.
the manager’s utility is thus

\[ U^M(\pi, e) \equiv F(0 | e)u(\pi(0)) + \int_0^\pi u(\pi(x)) f(x | e) \, dx - \psi(e). \]  

(1)

The first term reflects compensation if the firm becomes nonviable, while the second term reflects compensation when the firm is solvent. One might think, for example, of \( \pi(0) \) being the salary paid to the manager before the stock price is realized.

The incentive compatibility constraint is

\[ e \in \arg \max_{e' \in [0, \bar{e}]} U^M(\pi, e'). \]  

(IC)

For the compensation contracts \( \pi \) we will consider, there will be a unique effort level satisfying IC. We denote this effort level by \( e^*(\pi) \), writing simply \( e^* \) when \( \pi \) under consideration is clear.

A firm that grants stock or options as part of a new contract will usually not be able to revoke previously awarded securities. Nor can the base salary typically be negative. Thus, the prior stock and options holding of the manager serves as a lower bound on the payments to the manager under the new contract. More generally, a firm may face other constraints (legal, social, etc.) on how little the agent can be paid as a function of the stock price. This is modeled by assuming that there is a nondecreasing, continuous, and piecewise differentiable function \( m(x) \) such that

\[ \pi(x) \geq m(x) \quad \forall x. \]  

(M)

A base salary or limited liability would result in \( m(x) \) being constant. If there are nonrevokable prior stock or options holdings, then \( m(x) \) increases over the relevant range of stock prices.

The participation (individual rationality) constraint for the manager is of the form

\[ U^M(\pi, e^*) \geq u_0. \]  

(IR)

In the standard principal agent framework (such as Holmström, 1979), the IR constraint is always binding. Intuitively, if the IR constraint is not binding, one can decrease the utility of the manager by a small constant for any realization without changing her incentives. When payments to the manager are bounded from below, this is no longer true. If \( M \) is sufficiently tight, the IR constraint may cease to be relevant.

We think that IR does not bind in many cases of practical interest. An example is when a firm gives incentive pay to an existing manager working on salary. Another example may be the case of an entrenched manager or a manager with special skills appropriate for a specific firm. We also find it hard to imagine that the IR constraint explains the magnitude of expected compensation for most
CEOs.\textsuperscript{7} Thus, we consider mostly the case of a nonbinding \textit{IR} constraint. We discuss this briefly in Section 4.

The utility of the investors given contract $\pi(\cdot)$ and effort level $e$ is

$$U_I(\pi, e) = -F(0 \mid e)\pi(0) + \int_0^\bar{x} (x - \pi(x)) f(x \mid e) \, dx.$$  \hfill (2)

Investors choose $\pi(\cdot)$ and $e$ to maximize $U_I(\pi, e)$ subject to $IC, M$, and $IR$ (which we assume is not binding).

Recall that

$$U_M(\pi, e) \equiv u(\pi(0))F(0 \mid e) + \int_0^\bar{x} u(\pi(x))f(x \mid e) \, dx - \psi(e).$$

So,

$$\frac{\partial U_M(\pi, e)}{\partial e} = I(\pi, e) - \psi'(e),$$  \hfill (3)

where

$$I(\pi, e) = u(\pi(0))F_e(0 \mid e) + \int_0^\bar{x} u(\pi(x))f_e(x \mid e) \, dx.$$  \hfill (4)

Thus, $I(\pi, e)$ measures the marginal incentive to the manager (before effort costs) to expend extra effort.

Integrating by parts,

$$I(\pi, e) = -\int_0^\bar{x} u'(\pi(x))\pi'(x)F_e(x \mid e) \, dx.$$  \hfill (5)

Incentive intensity thus depends on how much utility is affected by changes in $x$ (this is given by $u'(\pi(x))\pi'(x)$) and by how much effort affects the probability of a stock price below $x$ is affected (this is given by $F_e(x \mid e)$).

It will be convenient to replace ($IC$) with its first-order condition. A sufficient but not necessary condition for the validity of this approach is to assume that there are decreasing marginal returns to effort as measured by the stock price distribution. Formally, \textit{Convexity of the Distribution Function (CDFC)} is satisfied if $F_e(x \mid e) \geq 0$. Recalling that the effect of effort is to lower $F(x \mid e)$, this indeed implies that successive increases in effort lower $F(x \mid e)$ less.\textsuperscript{8}

\textsuperscript{7} Grinstein and Hribar (2004) use mergers and acquisitions to provide evidence that managers can actually influence their compensation. This supports the claim that participation constraints may not bind.

\textsuperscript{8} Rogerson (1985) studies CDFC and its relation to the first-order approach. Jewitt (1988) provides different sufficient conditions for the first-order approach to be valid. When dealing with contracts that have an option-like flavor (as, e.g., in Section 4), there is a natural nonconcavity introduced, and the Jewitt conditions are not applicable. Our main result, on the dominance of stock by an option when there is no nonviability risk, does however hold under the Jewitt conditions, because the base contract (stock) does not have this nonconcavity.
Using the first-order approach, we would like to replace IC by

\[ I(\pi, e) - \psi'(e) = 0. \]  

\( (IC') \)

For this to be valid, we need to know that first-order conditions are in fact sufficient for optima. To see this, note that, using Equation (5),

\[ \frac{\partial}{\partial e} (I(\pi, e) - \psi'(e)) = -\int_{0}^{\bar{x}} u'(x) \pi'(x) F_{ee}(x | e) dx - \psi''(e). \]

By CDFC, \( F_{ee} \) is positive, while \( \psi'' \) and \( u' \) are positive by assumption. A sufficient condition for \( \frac{\partial}{\partial e} (I(\pi, e) - \psi'(e)) \leq 0 \) is thus that \( \pi'(x) \) is everywhere non-negative. We obtained:

**Lemma 1.** Suppose that CDFC is satisfied. For any given nondecreasing, piecewise differentiable contract \( \pi \), there exists a unique solution \( e^* \) to IC'. Hence, IC can be replaced with IC' and the first-order approach is valid.

In the rest of the article, we focus our attention on stock options and stock contracts. These result in nondecreasing and piecewise differentiable contracts. Thus, Lemma 1 allows us to use the first-order approach.

3. Stocks and Stock Options

In this section, we consider the case in which compensation contracts can take the form of either options or stock. Thus, \( \pi(x) \) takes the form

\[ \pi(x) = m(x) + \alpha \max(0, x - k), \]  

(6)

where \( m(x) \) represents the previous stock or options holdings of the manager and any base salary payment, \( \alpha \) is the proportion of the firm granted in new options, and \( k \in [0, \bar{x}] \) is the exercise price on these options. Thus, a contract can be represented by a pair \((\alpha, k)\). The case \( k = 0 \) corresponds to a stock.

Since prior compensation \( m(x) \) is composed of stocks and options only, it is continuous, piecewise linear, and (weakly) convex, and hence steepest at \( \bar{x} \). We assume that \( \alpha \in [0, 1 - m'(\bar{x})] \), so that the manager never owns more than the whole firm.

The payoff to the investors given \( \alpha, k, \) and \( e \) is (in a mild abuse of notation)

\[ U^I(\alpha, k, e) = \int_{0}^{\bar{x}} (x - \pi(x)) f(x | e) dx - m(0) F(0 | e). \]  

(7)

The payoff of the manager is

\[ U^M(\alpha, k, e) = u(m(0)) F(0 | e) + \int_{0}^{k} u(m(x)) f(x | e) dx \]

\[ + \int_{k}^{\bar{x}} u(m(x) + \alpha(x - k)) f(x | e) dx - \psi(e). \]
Note that Equation (4) becomes
\[
I(\alpha, k, e) = u(m(0))F_e(0 \mid e) + \int_0^k u(m(x))f_e(x \mid e)dx
+ \int_k^{\bar{x}} u(m(x) + \alpha(x - k))f_e(x \mid e)dx.
\]

Since \( \alpha \geq 0 \), and \( m'(x) \geq 0 \), \( \pi \) is non-decreasing, and so by Lemma 1 we can replace IC by its first-order condition \( I(\alpha, k, e) - \psi'(e) = 0 \).

Integration by parts yields
\[
I(\alpha, k, e) = -\int_0^k u'(m(x))m'(x)F_e(x \mid e)dx
- \int_k^{\bar{x}} u'(m(x) + \alpha(x - k))(m'(x) + \alpha)F_e(x \mid e)dx. \tag{8}
\]

Thus, to make the manager exert high effort, the contract should induce a high marginal utility in those regions of the distribution in which effort is most influential (where \( |F_e| \) is large).

Higher effort induces a better distribution of stock prices, but since the contract is non-decreasing it also induces a higher cost to investors. Thus, a priori it is not clear that the investors want the manager to exert higher effort. For a manager for whom both previously granted and current compensation is paid using stocks and options, \( x - \pi(x) \) is nondecreasing. This guarantees that the investors would always like the manager to work harder. In general:

**Lemma 2.** Assume that \( x - \pi(x) \) is nondecreasing. Then for any \( e \in [0, \bar{e}] \),
\[
\frac{\partial U^I}{\partial e} \geq 0.
\]

To see this, differentiate Equation (7) to obtain
\[
\frac{\partial U^I}{\partial e} = \int_0^{\bar{x}} (x - \pi(x))f_e(x \mid e)dx - m(0)F_e(0 \mid e)
= -\int_0^{\bar{x}} (1 - \pi'(x))F_e(x \mid e)dx \geq 0.
\]

The last equality follows from integration by parts and the fact that \( \pi(0) = m(0) \). The inequality follows since \( x - \pi(x) \) is nondecreasing and by FOSD. It is strict if \( x - \pi(x) \) is not a constant.

**3.1 Exercise price and incentives**

Figure 1 illustrates the effect of an increase in exercise price on the utility of the manager. The horizontal axis is the stock price, while the vertical axis is the utility of the manager. The figure shows the case of a constant minimum
The figure depicts the utility of a risk-averse manager with a minimum payment constraint of \( m(x) \equiv m \) compensated by an option. The horizontal axis is the stock price, and the vertical axis is the utility of the manager. The figure shows the effect of increasing the strike price from \( k_1 \) to \( k_2 \) on the utility of the manager.

payment \( m(x) \equiv m \). For stock prices less than \( k \), the manager always gets \( u(m) \), and thus does not care about changes in the stock price. We term \([0, k]\) the numb region. For values in \([k, \bar{x}]\), the utility of the manager is increasing and concave, reflecting the linearity of the contract over that region and the risk aversion of the manager.

When the exercise price increases from \( k_1 \) to \( k_2 \), there are two effects on the incentives of the manager. First, the numb region is enlarged and hence the manager does not care about changes in stock price over a larger region. But, for stock prices above \( k_2 \), the marginal utility of the manager is higher, and hence her incentives are increased: For all \( x \in [k_2, \bar{x}] \), \( u'(m + \alpha(x - k_1)) < u'(m + \alpha(x - k_2)) \).

To see this formally, note that the optimal effort level \( e^* \) is given implicitly by \( IC' \). The implicit function theorem then gives us

**Remark 1.** The sign of \( \frac{\partial e^*}{\partial k} \) is equal to the sign of \( \frac{\partial}{\partial k} I(\alpha, k, e^*) \).

---

By the implicit function theorem,

\[
\frac{\partial e^*}{\partial k} = \frac{\frac{\partial}{\partial k} (I - \psi'(e^*))}{\frac{\partial}{\partial e} (I - \psi'(e^*))}.
\]

By the agent’s second-order condition, the denominator is negative at \( e^* \). Hence, the sign of \( \frac{\partial e^*}{\partial k} \) is equal to the sign of

\[
\frac{\partial}{\partial k} (I - \psi'(e^*)) = \frac{\partial I}{\partial k}.
\]
Stocks or Options?

![Graph showing the effect of effort on the price distribution](image)

**Figure 2**

The effect of effort on the price distribution

The figure illustrates how the FOSD effect of increased effort depends on the presence of nonviability risk. The figure depicts the cumulative price distribution for two effort levels $e_H > e_L$ in two cases. The left graph illustrates a case where there is no nonviability risk; hence $F(0 \mid e_H) = F(0 \mid e_L)$. The right graph illustrates a case in which there is nonviability risk (an atom at 0), and this risk is affected by managerial effort.

Differentiation of Equation (8) then yields

$$\frac{\partial I}{\partial k} = \alpha u'(m(k))F_e(k \mid e) + \alpha \int_k^{\bar{x}} u''(m(x) + \alpha(x - k))(m'(x) + \alpha)F_e(x \mid e) \, dx.$$  \hspace{1cm} (9)

The first term is negative and represents the increase in the numb region. The second term is positive and reflects that the manager is “hungrier” over the interval $[k, \bar{x}]$. The effect of a change in the strike price on effort is the net effect of these two conflicting forces.

3.2 Stocks versus options: The role of nonviability risk

Equation (9) allows us to think systematically about when investors might prefer options and when they might prefer stock as a motivational tool. Our first observation is that if nonviability is not an issue (i.e., $F(0 \mid \cdot) \equiv 0$), then pure stock is never optimal. In particular, increasing the exercise price a little above zero induces a higher effort level by the manager at a lower cost. So, only if nonviability is important could compensation purely by stock be optimal. Healthy and stable firms should prefer options.

Figure 2 clarifies the intuition behind this result. It depicts cumulative distribution functions for two levels of effort, $e_H > e_L$. The left graph illustrates a case with no nonviability risk. Since $F(0 \mid \cdot) \equiv 0$, $F_e(0 \mid \cdot) = 0$ as well, and so, near zero, $\mid F_e \mid$ is small. But then by Equation (9), near $k = 0$, the damage...
of a small increase in the numb region on incentives is second order. There is a first-order improvement in incentives over the rest of the support. Hence, incentives are increased on net. The contract is of course cheaper for any given stock price. We obtain

**Proposition 1.** Suppose that \( F(0 \mid e) = 0 \) for all \( e \in [0, \bar{e}] \), so that there is no nonviability risk. Then, for any contract that involves granting pure stock, there is an option-based contract that pays the manager less at each stock price realization, but induces a strictly higher effort level.\(^{10}\)

The key observation here is that regardless of the existing compensation or stock holdings of the manager, lowering compensation to the minimum near a stock price of zero has a second-order effect on incentives if \( F_e(0 \mid e) \equiv 0 \) (the no-nonviability risk case) but improves incentives everywhere else. A corollary is

**Corollary 1.** Suppose that \( F(0 \mid e) = 0 \) for all \( e \in [0, \bar{e}] \), so that there is no nonviability risk. Then, granting stock to the manager is never optimal.

The point here is that under stock- and option-based compensation, compensation never rises faster than the total value of the firm.\(^{11}\) But then, the net payoff to the investors \( x - \pi(x) \) is nondecreasing, and so Lemma 2 applies. Since the contract pays less at each stock price realization, the investors are thus strictly better off.

When managerial effort does affect the probability of nonviability, as in the right graph of Figure 2, this conclusion need not follow. In particular,

\[
\left. \frac{\partial I(\alpha, k, e)}{\partial k} \right|_{k=0} = \alpha u'(m(0))F_e(0 \mid e) \\
+ \alpha \int_0^{\bar{x}} u''(m(x) + \alpha x)(m'(x) + \alpha)F_e(x \mid e) \, dx.
\]

While the second term is positive, the first term, which reflects nonviability avoidance, is negative. When nonviability risk is substantial, it is straightforward to construct examples in which the first term dominates, and in which a stock contract performs better than any option contract.\(^{12}\)

Finally, while we restrict \( \pi(\cdot) \) to consist of only stock or options with a single strike price, the same conclusion holds if we allow \( \pi(\cdot) \) to include options of various strike prices. Assume that nonviability risk is zero and that \( \pi(\cdot) \) contains any stock. Then, simply rolling all the other current compensation into \( m(\cdot)\),

---

10 Nonviability risk could be significant (\( F(0 \mid e) > 0 \)), but the effect of effort on nonviability risk is irrelevant (\( F_e(0 \mid e) = 0 \)). The proposition would also hold in this case.

11 In contrast, for example, to a situation with bonuses.

12 See Kadan and Swinkels (2006) for such an example.
Corollary 1 shows that the principal is better off replacing the stock component of $\pi(\cdot)$ by positively priced options.

### 3.3 Too rich too soon—Number of options and incentives

One might expect an increase in the number of options or stocks granted to a manager to increase effort. Interestingly, this need not be true: Granting a risk-averse manager too many options might make her quite rich at moderate stock prices. This lowers her responsiveness to stock price changes at higher levels, potentially by enough to actually decrease her optimal choice of effort.

To see the intuition, consider the case that $m(x) \equiv m$ and recall from Equation (5) that incentives are determined by

$$I(\pi, e) = -\int_{0}^{\pi} \frac{\partial}{\partial x} u(\pi(x)) F_e(x \mid e) \, dx.$$  

The derivative of the manager’s utility with respect to the stock price for a given contract $(\alpha, k)$ and price $x \geq k$ is

$$\frac{\partial}{\partial x} u(m + \alpha(x - k)) = \alpha u'(m + \alpha(x - k)).$$

If the manager is risk neutral, this increases in $\alpha$, but if the manager is risk-averse, the decrease in $u'$ may swamp the increase in $\alpha$. We obtain

**Proposition 2.** An increase in the number of granted options (or stocks) may damage incentives.

Thus, scaling up an ill-designed compensation contract can push a risk-averse manager to lower effort levels. Note again the critical role of the minimum payment constraint $M$. In this setting, increasing $\alpha$ has the unavoidable effect of making the manager rich, perhaps damaging her incentives at higher stock prices.

Figure 3 illustrates the effect of a change in the number of options granted to a mildly risk-averse manager. Figure 4 illustrates a more highly risk-averse manager. To the right of $y$, the increase in $\alpha$ is dominated by the (richer) manager’s lower marginal utility and incentives are weakened.

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13 For example, assume $f(x \mid e) = 1 + \frac{1}{2}(1 - 2x)(1 - 2e)$ for $x, e \in [0, 1]$, $\Psi(e) = 0.5e^2$, $m(x) \equiv 1$, and $u(w) = w^{1 - y}$. Then, $e^* = \alpha \int_{k}^{1} (1 + \alpha(x - k))^{-y} (x - x^2) \, dx$, and

$$\frac{\partial e^*}{\partial \alpha} = \int_{k}^{1} (1 + \alpha(x - k))^{-y} (1 - \frac{\alpha(x - k)}{1 + \alpha(x - k)}) \gamma(x - x^2) \, dx.$$  

Assume $\alpha = 1$, $k = 0$, and $\gamma = 5$. Substitution yields $\frac{\partial e^*}{\partial \alpha} = -\frac{1}{10} < 0$. From continuity, $\frac{\partial e^*}{\partial \alpha} < 0$ continues to hold for $\alpha$ strictly lower than 1 and $k$ strictly higher than 0.
The effect of the number of options on a mildly risk-averse manager
The figure illustrates the effect of increasing the number of granted options on the utility of a mildly risk-averse manager. The figure depicts the utility in two cases with $\alpha_2 > \alpha_1$. The low level of risk aversion implies that marginal utility is higher in the top utility curve than in the bottom curve at every stock price level above the strike price. In this case, incentives are strengthened by increasing the number of granted options.

The effect of the number of options on a highly-risk averse manager
The figure illustrates the effect of increasing the number of granted options on the utility of a highly risk-averse manager. The figure depicts the utility in two cases with $\alpha_2 > \alpha_1$. The high level of risk aversion implies that marginal utility in the case of $\alpha_2$ grants is lower compared to the case of $\alpha_1$ grants whenever the stock price exceeds $y$. In this case, an increase in the number of granted options may reduce incentives.
As suggested by Figure 3, the “too rich too soon” phenomenon cannot occur if the manager is only mildly risk-averse. Let the coefficient of relative risk aversion at \( w \) be \( \rho(w) \equiv -w \frac{u''(w)}{u'(w)} > 0. \) Then

**Proposition 3.** Assume \( m(x) \equiv m \geq 0. \) If \( \rho(w) \leq 1 \) for all \( w \) (with strict inequality somewhere), then \( \frac{\partial e^*(\alpha, k)}{\partial \alpha} > 0 \) for all \( \alpha \) and \( k. \)

### 3.4 Do options imply higher risk?

Beginning with Jensen and Meckling (1976), it has been argued that options induce managers to take more risk (since option value increases with volatility). Two recent papers suggest a more nuanced conclusion. Carpenter (2000) solves a dynamic investment problem of a risk-averse manager compensated with a call option. She shows that option compensation may or may not lead to greater risk seeking. Essentially, a risk-averse manager may respond to the increased exposure inherent in owning more options by choosing a safer strategy. Ross (2004) argues that the conventional “folklore” fails to account for the shape of the manager’s utility function. He argues (p. 224) that “It is routine for commentators to argue that call options increase the manager’s willingness to take risk. We now know, though, that this also depends on the wealth effect of the options...”

Carpenter and Ross consider project choice but not effort, which is the focus of our model. Of course, the choice of effort does indirectly affect the volatility of returns. The effect of options compensation on stock-price volatility in our setting is complicated by two effects. First, as we have shown, because of the same sorts of wealth effect that are central to Ross (2004), changes in option contracts can have counterintuitive effects on the choice of effort (as in Proposition 2). Second, increases in managerial effort may either increase or decrease the volatility of the stock price. For example, if extra effort primarily reduces bankruptcy risk, but does not affect the stock-price distribution conditional on survival, then extra effort reduces volatility. But, if extra effort means taking on new projects, then variance may well increase. So, as in Ross (2004) and Carpenter (2000), the relationship between incentives and volatility is highly ambiguous. Our results thus reinforce theirs.

### 4. Critical Prices and General Contracts

Our central result (and the one that we empirically test) is that for firms with no nonviability risk, options dominate pure stock. Given that essentially all real-world stock-based compensation consists solely of stock and options, this seems a highly relevant comparison. But, one could also ask the distinct question of

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14 Common figures used in asset-pricing calibration exercises are \( \rho > 3. \) Hence, this proposition should not be interpreted as saying that the phenomenon under discussion is rare.
whether something “option-like” arises in the absence of nonviability risk when the contract space is general.

We begin with a definition.

**Definition 1.** Returns to effort are single peaked if for each $e$, there is $x_e^*$ such that $F_e(x \mid e)$ is decreasing in $x$ for $x < x_e^*$ and increasing in $x$ for $x > x_e^*$.

So, $x_e^*$, the critical price, is the one at which the distribution over price is most responsive to effort. For a firm with no nonviability risk, and single peaked returns to effort, it must be that $x_e^* > 0$ (since $F_e(0 \mid e)$ is then 0). For healthy firms, one would expect $x_e^*$ to be considerable. On the other hand, if the probability of firm failure is both significant and affected by managerial effort, then it may well be that $|F_e(x \mid e)|$ is largest at 0, so that $x_e^* = 0$.

A primitive condition for returns to effort to be single peaked is the Monotone Likelihood Ratio Property (MLRP), which requires that

$$\frac{F_e(S \mid e)}{F(S \mid e)} < \frac{F_e(T \mid e)}{F(T \mid e)}$$

for any two intervals $S, T \subset [0, \bar{x}]$ where $S$ lies strictly to the left of $T$.15

**Lemma 3.** If MLRP is satisfied, then returns to effort are single peaked. If there is no nonviability risk, then $x_e^* > 0$.

Assume that MLRP holds and also assume that $\frac{f_e(x \mid e)}{f(x \mid e)}$ is bounded.16 Then, if the IR constraint is not binding at the optimum, the optimal contract is given implicitly by

$$\frac{1}{u'(\pi(x))} = \begin{cases} \max \left\{ \frac{1}{u'(m(x))}, \mu \frac{f_e(x \mid e)}{f(x \mid e)} \right\}, & x > 0 \\ \frac{1}{u'(m(0))}, & x = 0 \end{cases}$$

where $\mu > 0$ is the Lagrange multiplier on the IC constraint.17

An immediate implication of Equation (11) is that the optimal contract in the absence of nonviability risk is option-like: It specifies paying the minimum $m(x)$ for all $x$ up to a price level $\hat{k}$ (a “strike price”) that exceeds $x_e^*$. Formally,

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15 This generalizes the standard definition of MLRP to allow for distributions that contain atoms. Note that MLRP implies FOSD.

16 This is to rule out nonexistence issues along the lines of Mirrlees (1999).

17 See Holmström (1979) and Jewitt et al. (2007) for the details of characterization and existence.
Proposition 4. Suppose that the IR constraint is not binding at the optimal effort level \( e \). Then, there is \( \hat{k} > x^*_e \) such that the optimal contract pays \( m(x) \) for all \( x < \hat{k} \). If there is no nonviability risk, then \( \hat{k} > 0 \).

This follows immediately from Equation (11), since \( \mu \frac{f_e(x | e)}{f(x | e)} > \frac{1}{u'(m(x))} \) can only hold where \( f_e \) is strictly positive. For \( x \leq x^*_e \), \( F_e(x | e) \) is decreasing in \( x \) by MLRP and Lemma 3, and hence \( f_e(x | e) \leq 0 \).

In particular, note that when there is no nonviability risk, then by Lemma 3, \( x^*_e > 0 \), and hence \( \hat{k} > 0 \). So, as was the case when considering the choice between stocks and options, a firm with full latitude in choosing compensation will still choose to pay more than the minimum for low stock prices only when there is significant nonviability risk.\(^{18}\)

4.1 Start-ups and dinosaurs

Our results make a strong case for the use of stock (granting partial ownership) in motivating the managers of start-ups. For most start-ups, there is a significant chance that the firm fails, so that \( F(0 | e) \) is high. This makes it likely that \( x^*_e \) is near 0. At the time a start-up goes public, many of the hazards of having no viable product or securing early financing have already been overcome. So, one could well imagine that \( F(0 | e) \) is less affected by effort than before. Hence, a transition toward options makes sense.

Start-ups may also be more likely to face a binding IR constraint. For example, part of what fills out the IR constraint in a typical firm is the prospect of generous future compensation. In start-ups, managers may view these long-term benefits skeptically. When IR is binding, payments below \( x^*_e \) can be optimal, making stock a more sensible choice.

The same forces hold when attempting to turn around a foundering firm, as for example, one in Chapter 11. Here again, managerial effort focuses on whether the firm survives at all, so that \( x^*_e \) may well be close to 0.\(^{19}\) Here the IR constraint argument (i.e., retaining the employee) for payments below \( x^*_e \) is perhaps even stronger, because if the firm fails, the manager may suffer damage to her reputation as well. As the firm emerges from Chapter 11 and the threat of bankruptcy is reduced, a transition to options again becomes sensible.

5. Empirical Analysis

Our model suggests that firms with a higher nonviability risk will tend to grant their managers more restricted stock, while firms with a lower nonviability risk

\(^{18}\) Jewitt et al. (2007) show that under a convexity condition on the principal’s problem, the optimal contract will pay the minimum near \( x = 0 \) even if the IR constraint is binding, as long as the M constraint actually harms the principal.

\(^{19}\) That is, \( x^*_e \) may be low even relative to existing stock prices. A typical example would be TWA, which, shortly before its takeover by AA, had a stock price of around $1. Holders of this stock presumably thought of themselves as holding a lottery with a very large atom at zero and a small chance that the company (and its stock price) would at some point return to health.
will tend to grant options. In this section, we provide evidence supporting this prediction. In the absence of data on compensation in start-up companies, we restrict attention to bankruptcy risk, a specific type of nonviability risk. We hypothesize (and find) a positive correlation between bankruptcy risk and the use of stock in CEO compensation contracts.

5.1 Related empirical literature

Empirical literature on the determinants of executive pay is extensive. Smith and Watts (1992) find that firms with more growth opportunities have higher levels of executive compensation and greater use of stock option plans. They also show that large and nonregulated firms have higher levels of executive compensation. Gaver and Gaver (1993) find that growth firms pay higher levels of cash compensation to their executives. Controlling for firm size, they show that the incidence of bonus plans, performance plans, and restricted stock plans does not differ between growth and nongrowth firms. Kole (1997) shows that restricted stock is most prevalent among research-intensive firms and firms in innovative industries. Aggarwal and Samwick (1999) provide empirical evidence supporting the prediction that pay-performance sensitivity is decreasing in the variance of the firm’s performance. Barron and Waddell (2003) suggest a model in which the manager not only makes an effort choice, but also affects project selection. Their empirical evidence shows that executive rank is a major determinant of the extent of use of incentive pay.20

The relation between bankruptcy risk and compensation mix has been relatively unexplored.21 Gilson and Vetsuypens (1993) provide an empirical examination of compensation in financially distressed firms from 1981 to 1987. They show that after debt renegotiations, CEO compensation is strongly related to the stock price. They do not study the compensation mix between stocks and options. They do find that for some senior managers, compensation is tied directly to the successful resolution of bankruptcy. This is consistent with the idea of granting incentives at points where the effort of the manager affects the distribution of prices the most.22

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20 The literature is split on whether stock-based compensation can be attributed to an optimal compensation scheme. Smith and Watts (1992) and Mehran (1995), for instance, provide evidence supporting advocates of incentive-contract compensation. By contrast, several authors have criticized the contracting approach and assign at least a portion of managerial compensation to windfall, luck, or managerial intervention. See, for instance, Yermack (1995), Bertrand and Mullainathan (2001), Bebchuk and Fried (2003), and Garvey and Milbourn (2005).

21 Guay (1999) studies the relation of the mix between stock and stock option holdings and the sensitivity of CEOs’ wealth to equity risk. He finds that this sensitivity increases with stock options holding and provides evidence on cross-sectional determinants of convexity in executive incentive schemes. He does not, however, study the effect of bankruptcy on the compensation mix.

22 Another interesting finding of Gilson and Vetsuypens (1993) is that almost one third of the firms in their sample reset the exercise price of outstanding options that have fallen out of the money. Our model does not consider the costs and benefits of resetting executive stock options. These are captured in an elegant way in Acharya et al. (2000). In their model, there is some information revelation after the contracting stage, resulting in a value-enhancing role for exercise price resetting, even from an ex-ante point of view.
Bryan et al. (2000) examine the mix, incentive intensity, and economic determinants of stock-based compensation. They compare restricted stock and stock options. Although bankruptcy risk is not explored in their paper, it can be considered a first-cut test to our hypothesis, because they do look at capital structure. They show that firms with high leverage ratios tend to grant their managers more restricted stock both in terms of amount and in terms of incentive intensity. Bryan et al. find this result puzzling (pp. 684) since they do not expect stock-based incentive pay to be used in firms experiencing debt-related agency costs. Our model suggests an explanation: Restricted stock is an efficient way to motivate managers when bankruptcy risk is substantial. The goal of this section is to provide a deeper and more extensive test of this claim.

5.2 Data and methodology
We examine a panel of firms and regress measures of stock-based compensation mix on bankruptcy proxies. We use variables identified in the papers above as determinants of stock-based compensation as controls. This empirical approach is motivated by papers such as Yermack (1995) and Bryan et al. (2000).

The sample period is 1992–2004. Compensation data come from S&P ExecuComp. An observation is a payment by a firm to an executive in a given year. We restrict attention to CEOs only, so each firm has just one observation in each year. Since our purpose is to study the determinants of restricted stock versus stock options grants, we restrict attention to observations in which the firm granted its CEO either stock options or restricted stock. Financial statement data used for calculating proxies for bankruptcy risk are drawn from S&P Compustat. Return data are taken from CRSP. We use these data to calculate firms’ return volatility, an important control variable in our analysis, and to estimate the KMV bankruptcy measure discussed below.

Our dataset consists of 14,478 observations coming from 2418 different firms over 13 years. Table 1 provides descriptive statistics. Stock options are significantly more popular than restricted stock: The average stock option grant is $2.6 million compared to $475,000 of restricted stock. The median-restricted stock grant is zero.

We use three proxies for bankruptcy risk:

5.2.1 Altman’s Z-Score. This measure is defined (see Altman, 1968) as

\[ Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5, \]

23 Core and Guay (1999) offer an alternative approach. They first specify and estimate an empirical model for CEOs’ optimal portfolio holdings of equity incentives in year \( t - 1 \). The residuals from this model proxy for the deviation from optimal holdings. They then use these residuals as an explanatory variable when studying the determinants of equity grants at time \( t \). Recall that our theoretical model shows that our key prediction holds regardless of the current portfolio of the manager. In particular, our model suggests that when firms are more affected by bankruptcy considerations, they will tend to use restricted stock regardless of the form of prior compensation. This justifies our cross-sectional approach.

24 To avoid outliers, we removed 31 observations in which the CEO was granted more than $100 million in stock or options in 1 year. Our estimates are essentially unaffected by this.
### Table 1

**Summary statistics**

<table>
<thead>
<tr>
<th></th>
<th>Observations</th>
<th>Mean</th>
<th>Median</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base salary ($ thousands)</td>
<td>14,478</td>
<td>620</td>
<td>569</td>
<td>324</td>
</tr>
<tr>
<td>Restricted stock grants ($ thousands)</td>
<td>14,478</td>
<td>475</td>
<td>0</td>
<td>2,090</td>
</tr>
<tr>
<td>Stock options grants (B-S value in $ thousands)</td>
<td>14,442</td>
<td>2,581</td>
<td>921</td>
<td>5,410</td>
</tr>
<tr>
<td>Bonuses ($ thousands)</td>
<td>14,478</td>
<td>710</td>
<td>375</td>
<td>1,516</td>
</tr>
<tr>
<td>Net sales ($ millions)</td>
<td>14,457</td>
<td>4,650</td>
<td>1,279</td>
<td>12,155</td>
</tr>
<tr>
<td>Market to book value</td>
<td>14,351</td>
<td>3.8</td>
<td>2.2</td>
<td>51.2</td>
</tr>
<tr>
<td>Market value of equity ($ millions)</td>
<td>14,363</td>
<td>6,360</td>
<td>1,442</td>
<td>19,164</td>
</tr>
<tr>
<td>Assets ($ millions)</td>
<td>14,458</td>
<td>11,396</td>
<td>1,534</td>
<td>48,207</td>
</tr>
<tr>
<td>Yearly standard deviation of returns</td>
<td>14,027</td>
<td>0.438</td>
<td>0.375</td>
<td>0.239</td>
</tr>
<tr>
<td>Percentage of shares owned by the CEO (%)</td>
<td>6,603</td>
<td>4.06</td>
<td>1.38</td>
<td>6.87</td>
</tr>
<tr>
<td>Debt ratio (book)</td>
<td>14,167</td>
<td>0.57</td>
<td>0.58</td>
<td>0.21</td>
</tr>
<tr>
<td>Debt ratio (market)</td>
<td>14,077</td>
<td>0.41</td>
<td>0.38</td>
<td>0.24</td>
</tr>
<tr>
<td>Z-Score</td>
<td>11,172</td>
<td>4.55</td>
<td>3.27</td>
<td>4.50</td>
</tr>
<tr>
<td>KMV—Expected default probability</td>
<td>12,500</td>
<td>0.028</td>
<td>1.0E-9</td>
<td>0.12</td>
</tr>
<tr>
<td>S&amp;P Long-term debt rating</td>
<td>8,237</td>
<td>10.4</td>
<td>10</td>
<td>3.34</td>
</tr>
</tbody>
</table>

This table presents the number of observations, mean, median, and standard deviation of compensation and other characteristics of the data in our sample. Each observation is a yearly grant by a firm to its CEO. Each firm has just one observation in each year. The sample period is 1992–2004. Compensation data are taken from ExecuComp, whereas accounting and fiscal year-end market-value data are taken from Compustat. Yearly standard deviation of returns is the average daily standard deviation of returns (calculated using CRSP data) multiplied by $\sqrt{250}$.

where $X_1$ is net working capital scaled by assets, $X_2$ is retained earnings scaled by assets, $X_3$ is pretax earnings scaled by assets, $X_4$ is market value of equity scaled by book liabilities, and $X_5$ is sales scaled by assets.\(^{25}\) The average Z-Score in our sample is 4.55 (Table 1).

#### 5.2.2 KMV-Merton expected default probability.

This measure applies the framework of Merton (1974), where the equity of the firm is a call option on the underlying value of the firm with a strike price equal to the face value of debt. The model defines the distance to default as the number of standard deviations by which the market value of the firm exceeds the face value of debt. This number is plugged into a cumulative distribution function to calculate the probability of default for a specific horizon (see Duffie and Singleton, 2003). The difficulty is that both the total value of the firm and its volatility are unobservable. These are inferred from the value and volatility of equity, and several other observable variables by numerically solving a set of nonlinear simultaneous equations. We estimate the KMV-Merton expected default probabilities for a one-year horizon using the detailed algorithm given in Bharath and Shumway (2004), which is similar to the method used in Vassalou and Xing (2004). The average KMV-Merton, one-year horizon expected default probability in our sample is 2.4% (Table 1).

#### 5.2.3 Standard and Poor’s long-term credit rating.

S&P credit ratings range from AAA to D. The values of long-term debt ratings in Compustat (data item 280) range from 2 (for AAA) to 27 (for D). Thus, a higher score

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\(^{25}\) As is usual, for this measure we restrict attention to observations for which the Z-Score is non-negative.
corresponds to poorer creditworthiness. Credit ratings are only available for about 57% of our observations, lowering the power of tests using this measure. The average debt rating in our sample is 10.4, which corresponds to an S&P rating between BBB and BBB+.

5.3 Taxation and expensing
Our theoretical model does not consider the differences between stock options and restricted stock in terms of taxation or expensing. To test the model, one has to control for such differences. Note that accounting rules for expensing restricted stock and stock options, set by the Financial Accounting Standards Board (FASB), are separate from taxation rules, set by the IRS.

5.3.1 Taxation. In principle, there is no difference between the taxation of stock options and restricted stock from the point of view of either the manager or the firm. There is no tax event at the granting date, whereas the vesting date constitutes a tax event for both. On the vesting date, the manager has to pay taxes on the fair value of the proceeds from her stock/options. The firm then gets a tax deduction equal to that fair value.

The only difference between the taxation of these two compensation contracts lies in Section 83(b) of the Internal Revenue Code. This section allows the manager, within 30 days of receiving a restricted stock (but not for options), to be taxed on the grant year instead of the vesting year. If the employee elects to follow this route, then the firm must also deduct the fair value of the restricted stock during the grant year instead of the vesting year.

At the outset, it is not clear that a manager’s election to follow Section 83(b) either hurts or helps the firm. If the stock price goes up significantly between the grant date and the vesting date, then the firm might have been better off if the manager had not chosen to be taxed early. On the other hand, if the stock price falls or rises moderately between the two dates, then the firm is better off when getting the tax deduction early.

Although there is no public data on this issue, it appears that managers of publicly traded firms rarely choose to follow Section 83(b) since this imposes early taxation on them. However, it may be that the existence of this option creates a conception among financial managers that restricted stock provides less tax benefit to their firms. Thus, firms with relatively high marginal tax rates may be reluctant to use restricted stock.

To control for tax differences across firms, we use Graham’s Marginal Tax Rates (see Graham, 1996). These account for many important features of the tax code, including uncertainty about taxable income, deferred taxes, net operating loss carryforwards and carrybacks, and certain tax credits.

5.3.2 Expensing. In December 2004, the FASB changed the procedures for the expensing of employee stock options. As of 15 June 2005, public companies are required to report employee stock options as expenses on their
financial statements. The previous standard allowed companies to expense options granted at or above the money, but did not require this. Restricted stocks, however, are always in-the-money, and so were always expensed.

Thus, during our entire sample period, the accounting treatment of stock options was different from the treatment of restricted stock. A potential concern regarding an empirical study of the mix between stocks and options is that the results would be affected by these expensing considerations.

An interesting development in this area during the years 2002–2004 allows us to address this concern. In the wake of a number of accounting scandals that were tied to stock options, numerous companies voluntarily decided to expense executive stock options. This was presumably intended to demonstrate transparency and integrity in executive compensation. We obtained a list of all public companies that announced their intention to voluntarily expense executive stock options between 2002 and 2004 from Bear Stearns Inc. This hand-collected dataset consists of 483 companies of which 180 are covered in the Execucomp database. The dataset also specifies the year in which the firm started to expense the stock options.26

If the expensing treatment has an effect on the compensation contract used by corporations, then compensation contracts used by firms that expense stock options should systematically differ from contracts used by firms that do not expense. For example, firms that voluntarily expense stock options may be more likely to use restricted stock, since the fact that restricted stock is expensed is no longer a deterrent. The relevant question for this article is whether such a potential systematic relation between expensing and contract choice affects our results regarding the effect of bankruptcy risk.

To address this question we restrict some of our analysis to the years 2002–2004, when the vast majority of voluntary expensing took place. We then add to the regression models below a dummy, which is 1 if the firm has announced that it voluntarily expenses stock options on or before the relevant fiscal year in which the options or restricted stock in question were granted and zero otherwise. Restricting attention to these years reduces the number of firm-year observations in our panel to 3256 (22% of the original number). The ‘Expense’ dummy is 1 in 271 of these observations.

5.4 Results
To get a preliminary look into our results, we split the sample into two groups. In Group 1, with 4073 observations, firms made use of restricted stock to compensate their CEO in a specific year, while in Group 2, with 10,405 observations, they exclusively used stock options. Table 2 gives the mean, median, and standard deviation of debt and liquidity measures for the two groups.

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26 The complete expensing dataset is available upon request. Major corporations that voluntarily expended options between 2002 and 2004 include Coca-Cola, Daimler-Chrysler, DuPont, Exxon Mobil, GE, Home Depot, Nortel Networks, Pepsico, Procter & Gamble, SBC Communications, Target Corp, United Parcel Service, Verizon Communications, and Wal-Mart.
Table 2
Summary statistics of debt and liquidity measures by compensation type

<table>
<thead>
<tr>
<th></th>
<th>With restricted stock</th>
<th>With no restricted stock</th>
<th>% Difference in means</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N Mean Median STD</td>
<td>N Mean Median STD</td>
<td></td>
</tr>
<tr>
<td>Debt ratio (book)</td>
<td>4013 0.64 0.64 0.19</td>
<td>10,154 0.55 0.56 0.22</td>
<td>16.4%</td>
</tr>
<tr>
<td>Debt ratio (market)</td>
<td>3970 0.47 0.45 0.23</td>
<td>10,107 0.38 0.34 0.24</td>
<td>23.7%</td>
</tr>
<tr>
<td>Current ratio</td>
<td>3252 1.76 1.51 1.10</td>
<td>8,961 2.24 1.83 1.53</td>
<td>−21.4%</td>
</tr>
<tr>
<td>Quick ratio</td>
<td>3235 1.24 1.01 0.93</td>
<td>8,893 1.71 1.26 1.44</td>
<td>−27.5%</td>
</tr>
<tr>
<td>Inverse int. coverage ratio</td>
<td>3387 0.17 0.13 0.15</td>
<td>8,434 0.14 0.10 0.137</td>
<td>21.4%</td>
</tr>
</tbody>
</table>

This table presents the number of observations (N), average, median, and standard deviation for several measures of debt holdings. The sample period is 1992–2004. An observation is a grant by a company to a CEO in a specific year. The statistics are presented separately for companies that granted restricted stock and for companies that did not grant restricted stock. Debt ratio (book) is defined as total liabilities divided by assets. Debt ratio (market) is defined by total liabilities divided by the sum of total liabilities and the market value of equity. Current ratio is current assets divided by current liabilities. Quick ratio is current assets less inventories divided by current liabilities. The inverse of the interest coverage ratio is interest payments divided by operating income before depreciation. The rightmost column presents the percentage difference between the means in the cases with and without restricted stock grants. All the reported means are different at the 1% significance level except for the market debt ratio, which is different at the 5% level.

Companies in Group 1 have higher debt ratios and less favorable liquid asset positions. All differences in means are significant at the 1% level, except for market leverage, where significance is at the 5% level. For instance, the average debt ratio of companies that compensate using restricted stock is 64%, as compared to 55% for firms that do not. The average current ratio of companies that use restricted stock is 1.76 compared to 2.24 for companies that only use options. These results suggest that companies that are more likely to go bankrupt tend to use more restricted stock.

To conduct a formal multivariate test of our hypothesis, we employ two kinds of regression analyses. First are logistic regressions, which look at the likelihood that restricted stock is part of the CEO compensation package as a function of the bankruptcy measures. Second are OLS regressions relating the fraction of stock-based compensation that consists of restricted stock to bankruptcy risk.

Our logistic regressions take the form

\[
RST_{i,t} = \alpha + \beta Bankruptcy_{i,t} + \gamma_1 Vol_{i,t} + \gamma_2 Size_{i,t} \\
+ \gamma_3 MB_{i,t} + \gamma_4 DivYld_{i,t} + \gamma_5 RD_{i,t} + \gamma_6 ROA_{i,t} \\
+ \gamma_7 RET_{i,t} + \gamma_8 Expense_{i,t} + \gamma_9 Age_{i,t} + \gamma_10 Human\_cap \\
+ \gamma_{11} Ownership_{i,t} + \gamma_{12} MTR_{i,t} + \epsilon_{i,t},
\]

where

\[RST_{i,t}\] is a dummy equal to 1 if firm \(i\) used restricted stock in year \(t\) and zero otherwise.

\[Bankruptcy_{i,t}\] is the main explanatory variable. In each of the regressions, we use one of the three bankruptcy measures (Altman’s Z-Score, KMV-Merton, and S&P debt rating).
Vol_{i,t} = \text{the standard deviation of daily returns of the stock of company } i \\
\text{in fiscal year } t \text{ multiplied by } \sqrt{250}.
\[ \text{Size}_{i,t} = \log \text{ of book assets.} \]
MB_{i,t} = \text{market value of equity as of the last trading day of fiscal year } t \\
\text{divided by the book value of equity.}
\[ \text{DivYld}_{i,t} = \text{dividend per share divided by closing price for fiscal year } t. \]
RD_{i,t} = \text{research and development divided by book assets.}
\[ \text{ROA}_{i,t} = \text{pretax profits divided by assets.} \]
RET_{i,t} = \text{rate of stock return.}
\[ \text{Expense}_{i,t} = \text{a dummy equal to 1 if firm } i \text{ voluntarily expenses options in year } t \text{ (years 2002–2004 only).} \]
Age_{i,t} = \text{number of years since the firm first appeared on the CRSP tapes.}
\[ \text{Human cap}_{i,t} = \text{a dummy equal to 1 in human-capital-intensive firms: computer, electronics, and pharmaceutical firms.}^{27} \]
OWNERSHIP_{i,t} = \text{the percentage of CEO stock ownership.}
\[ \text{MTR}_{i,t} = \text{the after-financing Graham’s simulated marginal tax rate.}^{28} \]

The variable \textit{Bankruptcy}_{i,t} is the main explanatory variable in the regressions. We expect \( \beta \) to be negative for the Z-Score measure, and positive for the KMV-Merton and the S&P rating measures. A significant \( \beta \) of the right sign implies a positive relation between bankruptcy risk and the tendency of firms to use restricted stock. We also control for industry-fixed effects using the first digit of the SIC code (not reported).

For the OLS regressions, the one difference is that the dependent variable is now the fraction of the total value of stock-based compensation, which consists of restricted stock:

\[ RST_{\text{RATIO}}_{i,t} = \frac{RST_{\text{GRANTS}}_{i,t}}{RST_{\text{GRANTS}}_{i,t} + BLK_{VAL}_{i,t}}, \]

where \( RST_{\text{GRANTS}}_{i,t} \) is the total value of restricted stock grants to the CEO, and \( BLK_{VAL}_{i,t} \) is the total value of stock option grants to the CEO, calculated using the Black-Scholes formula.

Tables 3 and 4 report the results of the logistic and OLS regressions, respectively. Each table is divided into three sections, each dedicated to one of the three bankruptcy measures. For each bankruptcy measure, we divide the analysis into two. The first set of results uses the entire sample period (1992–2004), but does not control for the expensing decision. The second set of results restricts attention to the years 2002–2004, and controls for the expensing decision using the expense dummy. Since ownership data are available only for about 45% of the observations, regressions that include this variable suffer from

27 Specifically, we set this dummy to 1 when the first 3 digits of the SIC code are in 283, 357, 367, 481, 482, 483, 484, 489, 737.

28 Similar results obtain when we use the before-financing marginal tax rate.
Table 3
Logistic regressions

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<tr>
<td>Intercept</td>
<td>-1.282***</td>
<td>-1.059***</td>
<td>0.0848</td>
<td>1.1882</td>
<td>-1.621***</td>
<td>-1.553***</td>
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<tr>
<td>Bankruptcy</td>
<td>-0.070***</td>
<td>-0.071***</td>
<td>-0.127***</td>
<td>-0.149***</td>
<td>0.864***</td>
<td>0.937***</td>
</tr>
<tr>
<td>Volatility</td>
<td>-1.090***</td>
<td>-1.437***</td>
<td>-1.790***</td>
<td>-2.932***</td>
<td>-0.977***</td>
<td>-1.418***</td>
</tr>
<tr>
<td>Log assets</td>
<td>0.137***</td>
<td>0.151***</td>
<td>0.123***</td>
<td>0.0335</td>
<td>0.144***</td>
<td>0.180***</td>
</tr>
<tr>
<td>M/B</td>
<td>0.0000</td>
<td>-0.0005</td>
<td>-0.0001</td>
<td>-0.00039</td>
<td>-0.00061</td>
<td>-0.00406</td>
</tr>
<tr>
<td>Div. yield</td>
<td>0.0032</td>
<td>0.0032</td>
<td>0.0460</td>
<td>0.0945</td>
<td>0.00818</td>
<td>0.00827</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>-1.752***</td>
<td>-0.934***</td>
<td>-6.437***</td>
<td>-8.265***</td>
<td>-1.914***</td>
<td>-1.062</td>
</tr>
<tr>
<td>ROA</td>
<td>1.214***</td>
<td>1.257*</td>
<td>1.911*</td>
<td>1.028*</td>
<td>-0.0733</td>
<td>-0.3196</td>
</tr>
<tr>
<td>Mkt return</td>
<td>-0.00004</td>
<td>-0.00003</td>
<td>0.0020*</td>
<td>0.00235**</td>
<td>-0.0001</td>
<td>-0.00002</td>
</tr>
<tr>
<td>Expense</td>
<td>0.3705*</td>
<td>0.6899*</td>
<td>0.3075*</td>
<td>0.6899*</td>
<td>0.6456***</td>
<td>0.7952**</td>
</tr>
<tr>
<td>Age</td>
<td>0.0079***</td>
<td>0.0113***</td>
<td>-0.0046</td>
<td>-0.0037</td>
<td>0.0081***</td>
<td>0.0085***</td>
</tr>
<tr>
<td>Human_cap</td>
<td>-0.3632</td>
<td>-0.5041***</td>
<td>0.0984</td>
<td>0.2413</td>
<td>-0.5125***</td>
<td>-0.6552***</td>
</tr>
<tr>
<td>MTR</td>
<td>-0.3927***</td>
<td>-0.708***</td>
<td>-1.057***</td>
<td>-0.8966</td>
<td>-0.2223</td>
<td>-0.4879***</td>
</tr>
</tbody>
</table>

The dependent variable in all the regressions is a dummy variable equal to 1 if the company granted restricted stock to its CEO in a specific year and 0 if it did not. The models differ in the bankruptcy measure. Models (1), (2), (3), and (4) use the Z-Score. Models (5), (6), (7), and (8) use the KMV-Merton model, and modes (9), (10), (11), and (12) use the S&P long-term debt rating (Compustat item 280). Control variables are yearly volatility (average daily standard deviation of returns times $\sqrt{250}$), size (log of assets), market to book ratio (M/B), dividend yield, R&D (scaled by assets), pretax profits divided by assets (ROA), market return over the last year, firm age (number of years since first appearance on the CRSP tapes), a dummy for whether the firm is human-capital intensive (Human_cap), Graham’s after-financing marginal tax rate (MTR), percentage of stock ownership by the CEO (Ownership), and industry (first digit of SIC code, not tabulated). Models (3), (4), (7), (8), (11), and (12) are restricted to 2002–2004 and also control for whether the firm voluntarily expenses stock options (Expense dummy). We denote by *, **, and *** coefficients that are significant in the 10%, 5%, and 1% level, respectively, using Wald’s chi-squared test for logistic regressions.
Table 4
OLS regressions

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</tr>
<tr>
<td>Intercept</td>
<td>0.05228</td>
<td>-0.0582</td>
<td>0.2529*</td>
<td>-0.0058</td>
<td>0.1761**</td>
<td>-0.0893</td>
</tr>
<tr>
<td>Bankruptcy</td>
<td>-0.0024***</td>
<td>-0.0022***</td>
<td>-0.0071**</td>
<td>-0.0075***</td>
<td>0.1273**</td>
<td>0.0981***</td>
</tr>
<tr>
<td>Volatility</td>
<td>-0.1117***</td>
<td>-0.1005***</td>
<td>-0.1832**</td>
<td>-0.2512***</td>
<td>-0.1169**</td>
<td>-0.1013**</td>
</tr>
<tr>
<td>Log assets</td>
<td>0.0072**</td>
<td>0.0127**</td>
<td>0.014**</td>
<td>0.0113</td>
<td>0.0072**</td>
<td>0.0174**</td>
</tr>
<tr>
<td>M/B</td>
<td>-0.00001</td>
<td>-0.00002</td>
<td>-0.00002</td>
<td>-0.00003</td>
<td>-0.00003</td>
<td>-0.00004</td>
</tr>
<tr>
<td>Div. yield</td>
<td>0.00140*</td>
<td>0.00003</td>
<td>0.0124**</td>
<td>0.0228**</td>
<td>0.0023**</td>
<td>0.0012</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>-0.1151*</td>
<td>-0.0999</td>
<td>-0.5488***</td>
<td>-0.3904</td>
<td>-0.0549</td>
<td>-0.0732</td>
</tr>
<tr>
<td>ROA</td>
<td>-0.0375</td>
<td>-0.0365</td>
<td>-0.0963</td>
<td>-0.1726</td>
<td>-0.0249</td>
<td>-0.0511</td>
</tr>
<tr>
<td>Mkt return</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0003**</td>
<td>0.0002*</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Expense</td>
<td>0.1141*</td>
<td>0.0702</td>
<td>0.1218***</td>
<td>0.1314***</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Age</td>
<td>0.0013***</td>
<td>0.0015***</td>
<td>0.0001</td>
<td>0.0003</td>
<td>0.0011</td>
<td>0.0010***</td>
</tr>
<tr>
<td>Human_cap</td>
<td>-0.3310***</td>
<td>-0.0364***</td>
<td>-0.0092</td>
<td>0.0080</td>
<td>-0.0414**</td>
<td>-0.0438**</td>
</tr>
<tr>
<td>MTR</td>
<td>-0.0260</td>
<td>-0.04918*</td>
<td>-0.0737</td>
<td>-0.0470</td>
<td>-0.0092</td>
<td>-0.0291</td>
</tr>
<tr>
<td>Ownership</td>
<td>-0.1351**</td>
<td>-0.1649</td>
<td>-0.1351**</td>
<td>-0.0402</td>
<td>-0.0135</td>
<td>-0.0072</td>
</tr>
</tbody>
</table>

| N                    | 9429                       | 4310      | 1768                        | 877       | 9919                      | 4357      |
| Adj. R²              | 0.0571                     | 0.0585    | 0.0899                      | 0.0921    | 0.0490                    | 0.0507    |

The dependent variable in all the regressions is the proportion of restricted stock grants out of total stock-based compensation given by a firm to its CEO in a specific year. The models differ in the bankruptcy measure. Models (1), (2), (3), and (4) use the Z-Score; Models (5), (6), (7), and (8) use the KMV-Merton model; and models (9), (10), (11), and (12) use the S&P long-term debt rating (Compustat item 280). Control variables are yearly volatility (average daily standard deviation of returns times \( \sqrt{250} \)), size (log of assets), market to book ratio (M/B), dividend yield, R&D (scaled by assets), pretax profits divided by assets (ROA), market return over the last year, firm age (number of years since first appearance on the CRSP tapes), a dummy for whether the firm is human-capital intensive (Human_cap), Graham’s after-financing marginal tax rate (MTR), percentage of stock ownership by the CEO (Ownership), and industry (first digit of SIC code, not tabulated). Models (3), (4), (7), (8), (11), and (12) are restricted to 2002–2004 and also control for whether the firm voluntarily expenses stock options (Expense dummy). We denote by *, **, and *** coefficients that are significant in the 10%, 5%, and 1% level, respectively.
relatively low statistical power. Thus, in each case we report two specifications, one with and one without the ownership control variable.

The coefficients of the three bankruptcy measures have the right sign and are significant in 22 out of the 24 different specifications in Tables 3 and 4. Thus, consistent with the prediction of the model, a lower Z-Score, a higher KMV-Merton default probability, and a less favorable debt rating are all associated with a higher likelihood of restricted stock usage and with a higher proportion of restricted stock in the compensation mix.

It is interesting to consider some of our control variables. High volatility raises the probability of option compensation and the proportion of options in the compensation mix. This is consistent with the risk-shifting literature if we believe that the direction of causality goes from compensation to riskiness, so that managers who are compensated using stock options choose riskier projects. Size is a significant control variable—larger firms tend to compensate their managers using restricted stock. Firms that invest in research and development, human-capital-intensive firms, and young firms are more likely to use options. Managerial ownership is also highly significant: firms of which managers hold larger proportions tend toward options.

The expense dummy is positive in all of the specifications and significant in 9 out of the 12 specifications that include it. This suggests that, as expected, firms that expense stock options are more likely to use restricted stock. Finally, Graham’s marginal tax rate has a negative sign, and is significant in 6 out of the 24 specifications in Tables 3 and 4. This suggests that during our sample period, firms with higher tax rates were weakly reluctant to use restricted stock, consistent with the rationale presented in the previous section.

6. Conclusion

We study the problem of eliciting effort from a risk-averse manager when there are significant constraints on the minimum she can be paid. We consider how the choice of stocks versus options depends on the statistical structure of stock returns and, in particular, on the probability that the firm becomes nonviable.

The key trade-off given a lower bound on compensation is that increasing the slope of incentives at low stock prices raises wealth at higher stock prices. The risk-averse manager is thus less responsive to changes in income caused by changes in stock prices at these higher levels. Hence, there is a trade-off between providing incentives at one stock price level, and the ability to do so at higher stock prices.

Only firms with significant nonviability risk (e.g., financially distressed firms and start-ups) should use pure stock. This holds regardless of the existing portfolio of the manager.
We find an empirical relationship between proxies for bankruptcy risk and both the likelihood that firms use restricted stock, and the fraction of incentive pay granted in this manner, controlling for firm characteristics, previously granted stock-based compensation, corporate tax rates, and the decision by a firm whether to expense stock options.

Our theoretical results reinforce IBM’s attempt to improve motivation by increasing the exercise price of its stock options. They are also consistent with the choice of Conseco to use restricted stock while trying to emerge from bankruptcy. The results, however, suggest that Microsoft, which seems rather far from bankruptcy (having significant cash reserves), should not compensate its managers for low stock price realizations: Microsoft could both increase managerial performance and lower compensation costs by increasing the exercise price of its stock-based compensation.29

While we focus on a setting where effort is the only choice faced by the manager, the basic idea extends to any situation where an imperfectly monitored manager has different interests than investors. Managers may value expansion, being on the technical forefront, nice corporate jets, avoiding the pain of firing underperforming colleagues, or moving the corporate head office to a sunnier locale more than investors. Stock-based compensation helps to mitigate this misalignment. All else being equal, making the manager’s income responsive to stock price changes at low levels of the stock price improves the incentives of the manager to make appropriate choices. The difficulty is that doing so may make the manager very wealthy at more normal stock prices. And a wealthy manager may well, at the margin, value a nicer corporate head office over cash. So there is the same trade-off between providing incentives at one stock price level and the ability to do so at higher stock prices.

In our model, the stock price is the only indicator of firm performance. The key points of our analysis would, however, also apply to rewards based on performance indicators such as earnings or sales growth. In particular, making the manager’s pay more sensitive to changes in any given indicator at low levels makes it more difficult to provide strong incentives at higher levels of the performance indicator.

Appendix: Proofs

**Proof of Proposition 1:** Consider a stock contract \((\alpha, k)\), where \(\alpha > 0\) and \(k = 0\). Obviously, a small increase in \(k\) keeping \(\alpha\) constant lowers the payment to the manager at every price realization. We will show that it increases managerial effort.

By assumption, \(F(0 \mid e) \equiv 0\), implying that \(F_e(0 \mid e) \equiv 0\) as well. Thus, from Equation (9),

\[
\frac{\partial I}{\partial k} \bigg|_{k=0} = \alpha \int_0^\bar{x} u''(m(x) + \alpha x))(m'(x) + \alpha)F_e(x \mid e) \, dx > 0,
\]

29 Microsoft does tie the amount of restricted stock granted to its executives to several performance measures, which helps incentives.
where the inequality follows from risk aversion of the manager, first-order stochastic dominance, \( \alpha > 0 \) and \( m' \geq 0 \). Thus, by Remark 1, \( \frac{\partial e^*}{\partial k} \bigg|_{k=0} > 0 \). ■

**Proof of Corollary 1:** Assume that there is an optimal contract \((\alpha, k)\), where \( \alpha > 0 \) and \( k = 0 \). Note that

\[
\frac{dU_I}{dk} \bigg|_{k=0} = \frac{\partial U_I}{\partial k} \bigg|_{k=0} + \frac{\partial U_I}{\partial e} \frac{\partial e^*}{\partial k} \bigg|_{k=0}.
\] (12)

From Proposition 1, \( \frac{\partial U_I}{\partial k} \bigg|_{k=0} > 0 \) and \( \frac{\partial e^*}{\partial k} \bigg|_{k=0} > 0 \). By assumption \( \alpha \in [0, 1 - m'(x)] \), and hence (from the convexity of \( m(x) \)) \( m'(x) + \alpha \leq 1 \) for all \( x \). Consequently, \( \pi'(x) \leq 1 \) for all \( x \), and so by Lemma 2, \( \frac{\partial U_I}{\partial e} \geq 0 \). Thus, \( \frac{dU_I}{dk} \bigg|_{k=0} > 0 \), a contradiction. ■

**Proof of Proposition 3:** As in Remark 1, the sign of \( \frac{\partial e^*}{\partial w} \) is equal to the sign of \( \frac{\partial I}{\partial w} \). By Equation (8),

\[
\frac{\partial I}{\partial \alpha} = -\int_k^\bar{x} u'(m + \alpha(x - k))F_e(x \mid e) \, dx
\]

\[
-\alpha \int_k^\bar{x} (x - k)u''(m + \alpha(x - k))F_e(x \mid e) \, dx
\]

\[
= -\int_k^\bar{x} u'(m + \alpha(x - k)) \left[ 1 - \frac{\alpha(x - k)}{m + \alpha(x - k)} \rho(m + \alpha(x - k)) \right]
\]

\[\times F_e(x \mid e) \, dx.\]

Using \( \rho(w) \leq 1 \) and \( m \geq 0 \), the bracketed term is everywhere non-negative and is positive where \( \rho(w) < 1 \). Since \( u \) is twice continuously differentiable, \( \rho(w) < 1 \) for some interval of positive length. Since \( F_e < 0 \) on \((0, 1)\), we are done. ■

**Proof of Lemma 3:** Since we allow for an atom only at zero, we have for all \( x \)

\[
F_e(x \mid e) = \frac{\partial F(x \mid e)}{\partial e} = \frac{\partial}{\partial e} \left[ F(0 \mid e) + \int_0^x f(z \mid e) \, dz \right]
\]

\[= F_e(0 \mid e) + \int_0^x f_e(z \mid e) \, dz.\]

Therefore, for all \( x > 0 \),

\[
\frac{\partial F_e(x \mid e)}{\partial x} = f_e(x \mid e).
\]

By MLRP and since there are no atoms for \( x > 0 \), \( \frac{f_e(x \mid e)}{f(x \mid e)} \) is strictly increasing for \( x > 0 \). But, \( f_e(x \mid e) \) has the same sign as \( \frac{f_e(x \mid e)}{f(x \mid e)} \). Thus, \( f_e(x \mid e) = 0 \) at most once. Further,

\[
\int_0^\bar{x} f_e(x \mid e) \, dx = -F_e(0 \mid e) \geq 0.
\]
Hence, $f_e(\bar{x} \mid e) > 0$. If $f_e(x' \mid e) = 0$ for some $x' \in (0, \bar{x})$, then define $x^*_e = x'$. Otherwise, define $x^*_e = 0$ (this can only happen if $-F_e(0 \mid e) > 0$ and so nonviability risk exists). Then, since $f_e(x \mid e) = 0$ at most once, clearly $\frac{\partial F_e}{\partial x} < 0$ to the left of $x^*_e$ and $\frac{\partial F_e}{\partial x} > 0$ to the right. So, $|F_e|$ is single peaked and maximized at $x^*_e$.

References


