The Problematic Delta Test for Dividend Equivalents

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Prop. reg. section 1.871-15 suggests a protocol for ensuring that U.S.-source dividends embedded in specified derivative instruments are subject to withholding tax. This article highlights problems with the use of delta in prop. reg. section 1.871-15 and suggests how the regulations might be revised. Although the alternatives we discuss have some problems, substantial improvements are possible, and we describe how they might best be achieved.

The proposed regulations define delta as “the ratio of the change in the fair market value of [a contract] to the change in the fair market value of the property referenced by the [contract],” with the denominator being calculated as the change in value of the number of shares “referenced” by an instrument. An instrument is subject to the regulations only if the delta is at least 0.7 when the long party acquires the contract. This definition of delta can be decomposed into two factors in the following way:

$$\delta = \frac{\text{Change in Contract Price}}{\text{Change in Price of a Single Share}} \times \frac{1}{\text{Number of Referenced Shares}}$$

The first factor is the change in the fair market value of the contract per change in the price of a single share. We call this the number of “implicit shares” because it is the number of shares in the replicating portfolio for the instrument, that is, the number of shares that would be required to hedge the instrument’s sensitivity to fluctuations in stock price. The second factor is simply the reciprocal of the number of referenced shares. The number of referenced shares can be difficult to ascertain in particular situations; examples are offered in a recent New York State Bar Association Tax Section article. We provide examples that go further and show that the number is not always well defined, in the sense that economically identical instruments can appear to reference different numbers of shares simply because the packaging and labeling of the instruments is superficially different.

We suggest that, at a minimum, the regulations be revised so that they do not depend on a particular number of referenced shares. There are two specific places where changes must be made. First,
the number of referenced shares should not be used in the calculation of dividend equivalent amounts in prop. reg. section 1.871-15(i)(1)(ii). This is easy to accomplish. The dividend equivalent amount is defined as the per-share dividend multiplied by the product of delta and the number of referenced shares. This latter product appears to depend on the number of referenced shares. However, the product is independent of that number because — as the decomposition into factors above shows — the delta defined in the proposed regs is proportionate to the reciprocal of the number of referenced shares. Accordingly, the product of delta and the number of referenced shares is just equal to the number of implicit shares. The dividend equivalent amount can be redefined simply as the per-share dividend amount multiplied by the number of implicit shares. The end result is the same, but without the need to determine the number of shares referenced.

The second change needed to eliminate the number of referenced shares is more difficult because it involves a substantive replacement for the 0.7 delta threshold test in prop. reg. section 1.871-15(d) and (e). We propose a test based on the number of implicit shares. The new test simply computes the ratio of the price of the number of implicit shares to the price of the instrument and determines whether this ratio is above a specified threshold, such as 0.7. This is the same as determining whether the elasticity of the instrument price with respect to stock price is above the specified threshold, with elasticity defined as the ratio of percentage changes in price.

Even with this new test, game-playing is possible if the test does not consider additional related instruments, or if elasticity can be made to change substantially after a one-time test at acquisition. We present further examples that illustrate those issues, and we suggest two specific additional safeguards. The first is a backstop test that uses a dollar-based threshold to identify instruments with a substantial amount of implicit shares, even if the amount is small relative to the price of the instrument. The second is periodic retesting of an instrument to detect whether it has changed its implicit share content over time. Although this latter test is conceptually necessary, it may be administratively unworkable. If so, it may be appropriate to have an antiabuse rule aimed at addressing situations in which strategies are being used to manipulate implicit share content before and after testing.

### Simple Forward Contracts and Example 1

We present below an example of problems with the delta-based test using simple forward contracts. First, we introduce notation and review the basics of how forwards work. Write $S$ for a share of stock in some particular company and $S_t$ for the price of $S$ at time $t$. We assume that $S$ pays a dividend once each year, at times $t = 1, 2, 3, \ldots$, and that the dividend is always a constant fraction $q$ of the value of $S$ just before the dividend is paid. When $t$ is a whole number, we adopt the convention that $S_t$ is the value of the stock immediately after the dividend at time $t$ is paid, so that $S_t/(1 - q)$ is the value of the stock immediately before the dividend and $qS_t/(1 - q)$ is the amount of the dividend. We assume that a constant risk-free rate, $r$, compounded annually, is available to all borrowers and investors.

The forward price of $S$ at time $t$ is defined as $F_t = (1 - q)S_t$. A forward to buy $S$ at time $t$ is a contract that requires the long party to buy a share of $S$ at time $t$ for an amount equal to $F_t$, which is denominated in dollars at the future date $t$. The value in today’s dollars is the present value, $PV[F_t] = (1 - q)S_t$. The short party takes the opposite position and must sell a share of $S$ at time $t$ for an amount equal to $F_t$. The value of $F_t$ is chosen so that it is costless for the parties to enter into the forward contract, and no payments are made by either party until time $t$. For simplicity, we assume that there are no transaction costs and that neither party is required to post any collateral.

To better understand the nature of a forward contract, recall why the specified value of $F_t$ makes the forward contract costless. That standard result in finance theory follows from the observation that the forward contract can be replicated — and hence replaced — by some transactions in the underlying stock, as well as appropriate risk-free borrowing and lending. Because the replicating strategy is costless, so is the equivalent forward. The specific replicating strategy for the long party to the forward is to (1) borrow $(1 - q)S_t$ dollars today at the risk-free rate, and invest the proceeds in $(1 - q)S_t$ shares of $S$; and (2) allow interest on the borrowing to compound, and immediately reinvest all dividends paid on the $S$ stock position into more $S$ stock. At time $t$, the total amount due on the

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*To see the practical significance of this focus on implicit shares, consider a straddle-like instrument that has the same economics as a purchased call on 1 million shares combined with a purchased put on 1.1 million shares. It is not clear what the correct number of referenced shares is for this instrument. However, the number of implicit shares is a clearly defined quantity, and equal to the delta of the instrument with respect to a single share. In this case, because the call and put are offsetting positions with respect to the underlying stock, the number of implicit shares may be relatively small, even though each individual option references a large number of shares.*

*A special additional rule is needed in case the price of the instrument is non-positive; the details are described in the principal proposals section below.*
borrowing will be exactly \( F_t \), and the \( S \) stock position will have a value equal to exactly \( S_t \) dollars, the price of one share of \( S \) at time \( t \). This reproduces exactly the economics of the long forward position.

Replication of the short position in a forward contract is similar. The steps that must be implemented are complementary to those of the long replication strategy, namely, (1) sell \((1 - q)^t\) shares of \( S \) short today, and invest the proceeds at the risk-free rate; and (2) finance payments due on the short sale when dividends are paid on the stock by shorting additional stock, and allow the investment at the risk-free rate to compound. At time \( t \), the total amount due on the short sales will be exactly \( S_t \) dollars, the price of one share of stock at time \( t \), and the total proceeds from the investment at the risk-free rate will be \( F_t \). This is an exact reproduction of the economics of the short forward position.

**Proposition 1:** The delta today with respect to a single share of \( S \) of a long forward contract for delivery of \( S \) in \( t \) years is \( \Delta_t = (1 - q)^t \). With respect to \( n \) shares of \( S \), the delta today of the same forward contract is \( \Delta_n = (1 - q)^t / n \). We have used a subscript on the symbol for delta to emphasize the number of shares with respect to which delta is calculated.

The proof of the proposition is straightforward. As of today, the long forward contract can be perfectly replicated by a long position in \((1 - q)^t\) shares of stock and riskless borrowing. The riskless borrowing has no sensitivity to changes in stock price, and so the delta of the forward is the same as the delta of the long stock position, which is just equal to the number of shares in that long position in the case of the calculation of \( \Delta_t \). The calculation of \( \Delta_n \) is then easy. We must always have \( \Delta_t = \Delta_n / n \), because the change in value of \( n \) shares of \( S \) is always \( n \) times the change in value of one share of \( S \).

**Proposition 2:** After \( u \) years have passed, the delta with respect to a single share of \( S \) of the forward contract described in Proposition 1, is \( \Delta_t = (1 - q)^{t-u} \).

The proof of Proposition 2 follows from the fact that every year, the number of shares in the replicating portfolio increases by a factor of \( 1/(1 - q) \) as a result of the reinvestment of dividends paid through that time. After \( u \) years, the replicating strategy described above will consist of a long position in \((1 - q)^{u-t}\) shares of \( S \). Table 1 details how the \( \Delta_t \) value described in Proposition 2 changes over time for some specific values of \( q \) and \( u \) when \( t = 20 \).

As the table makes clear, the value of delta can change dramatically over time, even for a simple forward contract. Consider, for example, an investor who enters into the long position on a 20-year forward on a stock that has a dividend payout rate of \( q = 2 \) percent. At the time of acquisition, the delta with respect to the stock is 0.67, and so the contract does not meet the 0.7 threshold of prop. reg. section 1.871-15. Just five years later, however, the contract would meet the 0.7 threshold, and as time passes, the delta increases and tends to 1 as the contract reaches its maturity.

**Proposition 3:** Consider a forward contract like that in Propositions 1 and 2, except that it provides for purchase of \( N \) shares of \( S \), instead of just one share, in \( t \) years for an amount \( NF_t \). No matter what the value of \( N \), we have \( \Delta_{PR} = \Delta_1 = (1 - q)^t \), where \( \Delta_{PR} \) denotes delta with respect to the number of referenced shares within the meaning of prop. reg. section 1.871-15. The delta of the proposed regulations for a forward is thus independent of the scale of the position in the underlying stock.

To prove the proposition, note that the number of shares referenced by the forward contract for purposes of prop. reg. section 1.871-15 is \( N \). Both the change in the value of the forward and the change in the value of the referenced stock are \( N \) times greater than they would be if only one share were at issue. As a result, the ratio of these two changes does not vary with \( N \) and is always the same as \( \Delta_1 \).

**Example 1:** Consider two separate investors, \( U \), who enters into a long position on a forward contract to purchase one share of \( S \) in \( u \) years, and \( T \), who enters into a long position on a forward contract to purchase \((1 - q)^{t-u}\) shares of \( S \) in \( t > u \) years but plans to sell the contract after \( u \) years. Both investors have exactly the same economic positions on a pretax basis, but the delta of the instrument held by \( U \) at the moment of acquisition is \((1 - q)^u \) and that of the instrument held by \( T \) is \((1 - q)^t \). If \( u \) is relatively small and \( t \) is relatively large, \( U \)'s delta may be above 0.7, while \( T \)'s delta may be below 0.7.

Table 2 provides further details about the instruments in the example. The middle two columns of the first and third rows of the table show that the replicating portfolios for \( U \) and \( T \) have identical numbers of shares in them at both time 0 and time \( u \). For each forward, the number of shares in the

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8It is straightforward to check that the debt portions of the portfolios are also identical. For \( U \), the initial amount due on the (Footnote continued on next page.)
replicating portfolio at a given time is equal to the value of \( \Delta_1 \) at that time, and this can be determined from propositions 1 and 2. The value of delta under the proposed regulations can be determined from Proposition 3.

Compounding Forward Contracts and Example 2

Now consider an alternative type of forward contract under which a final payment is due to the long party in an amount equal to the value that would accrue if a single share \( S \) were purchased today, and if all dividends paid on the stock were immediately reinvested in more stock. This final payment is the same as \((1 - q)^t S_0\), where \( S_0 \) is the price of the stock at time \( t \) and the multiplier accounts for the reinvestment of dividends. The price owed by the long party at maturity is \((1 - q)^t F_t\), with this amount chosen so that the contract is costless to enter into. We call this a “compounding forward.” It is like a total return swap on a single share of stock because all returns on the share of stock are reflected in the final payment under the contract.

**Proposition 4:** Under prop. reg. section 1.871-15, the number of shares referenced by the compounding forward contract described above appears to be 1, and the delta of the long contract with respect to the referenced shares is \( \Delta_{PR} = 1.00 \). This value is independent of the maturity \( t \) of the contract and also independent of the dividend payout ratio \( q \).

To prove the proposition, note that the compounding forward contract can be replicated by a strategy that involves (1) borrowing \( S_0 \) dollars at the risk-free rate and investing in one share of \( S \); and (2) allowing the interest on the borrowing to compound and reinvesting all dividends paid on the stock in additional shares of \( S \). The delta of the long forward contract today is thus the same as the delta of a single share of stock, and assuming that the number of shares referenced is deemed to be 1 for purposes of the proposed regulations, it follows that \( \Delta_{PR} = 1 \), regardless of the values of \( t \) or \( q \).

**Example 2:** Consider two separate investors, \( P \), who purchases a plain forward contract for \((1 - q)^t S \) shares of \( S \) in \( t \) years, and \( R \), who purchases a compounding forward contract on a single initial share of \( S \) stock that will reinvest all dividends for a period of \( t \) years. The final payments under the two contracts are identical. Nevertheless, under prop. reg. section 1.871-15, \( P \)’s instrument has an initial delta of \((1 - q)^t \) and \( R \)’s instrument has an initial delta of 1. Thus, if \( q \) and \( t \) are sufficiently large, \( P \)’s instrument will not exceed the 0.7-delta threshold, but \( R \)’s instrument will.

The proposition follows immediately from propositions 3 and 4. Table 3 provides details about the instruments in the example. The middle two columns of the second and third rows of the table show that the replicating portfolios for \( P \) and \( R \) have identical numbers of shares at both time 0 and time \( t \).9 The value of delta under the proposed regulations can be determined from propositions 3 and 4.

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9It is straightforward to check that the debt portions of the portfolios are also identical. For \( P \), the initial amount due on the debt is \((1 - q)^t((1 + r)^t F_t = S_0 \), and for \( R \), the initial principal amount of debt is also \( S_0 \). Because these amounts are the same, and because all debt compounds at the same risk-free rate \( r \), the debt portions of \( P \)’s and \( R \)’s portfolios are the same at all times.
Discussion of Examples 1 and 2

The above examples can be made more concrete by considering how they apply in a particular situation. Consider a stock that pays dividends at a constant rate $q = 2$ percent. Suppose the investors from Example 1 engage in the following strategies: U purchases a five-year forward on one share of stock and holds it for five years, and T purchases a 20-year forward on 1.35 shares of stock and sells the forward after five years. The two investors have identical economic positions throughout the entire five-year period in question. Under the proposed regulations, however, the initial deltas for U and T are 0.9 and 0.67, respectively, meaning that U will be deemed to receive dividend equivalent amounts while T will not.

In the case of Example 2, consider again a stock with $q = 2$ percent, and suppose the investors engage in the following strategies: R purchases a 20-year compounding forward on one share of stock, and P purchases a 20-year (plain) forward on 1.49 shares of stock, with each holding his position for 20 years. The two investors have identical economic positions throughout the entire 20-year period. Under the proposed regulations, however, the initial deltas for R and P are 1 and 0.67, respectively, meaning that R will be deemed to receive dividend equivalent amounts while P will not.

The examples both show that it is possible to achieve results that are economically identical on a pretax basis but have opposite results on the delta test in the proposed regulations. Results turn on the “packaging” of an instrument, rather than its true economic nature.

It is worth examining what principles make the examples work. In each case, an implicit fungibility of current stock positions, future stock positions, and future dividend payments is being exploited. A forward contract can be replicated by a current position in stock, along with risk-free borrowing; a forward contract of one maturity can be replicated by one of another maturity, along with risk-free borrowing; and a future dividend payment can be replicated with either a current position in stock or with forward contracts, along with risk-free borrowing. It may seem natural to think of dividends and forward prices as fundamentally different from current positions in stock, but in fact all three are, in some sense, just reformulations of each other.

The examples rely on the assumption that dividends are paid at a constant rate $q$. Of course it is unrealistic to think that all dividends are equal to exactly the same percentage of stock price at all times. In some cases, however, an investor might expect payments to fall within a relative small range of percentages. More importantly, there are at least two games investors can play to achieve tax outcomes associated with proportional dividends. First, they can use plausible pricing models that assume proportional dividends in the process of calculating delta. Second, if there is investor demand, firms could issue classes of stock that pay proportional dividends and that could be used to evade the regulations under section 871(m) using ideas, for example, similar to those underlying Examples 1 and 2.

If the relationship between dividends and stock price is nonlinear, investors may still be able to take advantage of effective fungibilities between stock positions and dividends. In those cases, a future dividend may be viewed as a contingent claim on stock price, and this claim may be able to be replicated, or delta-hedged, with dynamic positions in the underlying stock. Doing this precisely can be complex, and its feasibility depends on the nature of stock price dynamics. Even when exact replication is very difficult, however, investors may be able to identify strategies that accomplish approximately what they desire.

Another ingredient found in both examples is the incoherence of the concept of the number of referenced shares. It is important in Example 1, for instance, that the number of referenced shares in U’s instrument is 1, while the number of referenced shares in T’s instrument is $1 - \frac{u-t}{u}$, even though the two instruments are economically identical on a pretax basis. Despite this economic equivalence, however, it seems that these numbers are natural interpretations of the meaning of the number of referenced shares in each case.

Special rules could be crafted to deal with the specific problems in the examples presented. For example, the numbers of referenced shares could be specially designated for some of the instruments involved to be something other than the numbers we call “natural.” These examples, however, are just illustrative of the general problem that the concept of a referenced number of shares is something that

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10This assumes the referenced number of shares in U’s and T’s forwards are taken to be 1 and 1.35, respectively.

11This assumes the referenced number of shares in R’s and P’s forwards are taken to be 1 and 1.49, respectively.
turns on labeling rather than fundamental economics. Even if special rules are created to deal with particular instances of problems, the fundamental incoherence of the concept of a referenced number of shares will remain and will likely be exploitable by still other instruments.

Principal Proposals
We have two thoughts about how to improve the proposed regulations. First, the definition of the dividend equivalent amount in prop. reg. section 1.871-15(i) should be restated to eliminate the concept of the number of shares referenced by an instrument. The product of the number of shares specified in prop. reg. section 1.871-15(i)(1)(ii)(A)(2) and the value of delta specified in prop. reg. section 1.871-15(i)(1)(ii)(A)(3) appear superficially to depend on the referenced number of shares because each term in the product depends on this number. However, the first term in the product is proportional to the number, while the second term is proportional to its reciprocal. As a result, the product is independent of the number of shares referenced, and so the number is unnecessary for the computation.

We propose specifically that the product of the two terms in prop. reg. section 1.871-15(i)(1)(ii)(A)(2) and (3) be replaced simply with $\Delta$, the delta for a single share of stock, that is, the ratio of the change in value of an instrument to the corresponding change in value of a single share of stock. To keep a strict parallel with the proposed regulations, we should multiply $\Delta$ by 1, the number of shares of stock with respect to which the delta value is computed, but because this additional multiplication by 1 changes nothing, we ignore it. We term this new quantity the number of implicit shares of stock in the instrument because it is the number of shares in the replicating portfolio for the instrument, that is, the number of shares that would be required to hedge the sensitivity of the instrument to fluctuations in stock price. The result using the number of implicit shares will be the same as that currently contemplated by the proposed regulations, but the cumbersome and incoherent concept of the referenced number of shares will be eliminated.

Our second proposal is to replace the 0.7-delta threshold test with a test that depends only on the economics of an instrument. In particular, we propose a new test that does not produce different results for economically identical instruments that happen to be labeled differently.

The theoretically ideal method would determine the number of implicit shares in an instrument whenever dividends are paid on the reference stock. This should be done for every instrument that references the stock. In practice, we recognize that it may be administratively necessary to use a one-time test at acquisition to identify only specific instruments for dividend equivalent treatment. A test checking whether the dollar value of implicit shares exceeds a threshold at acquisition could be used for this purpose, but it would miss smaller instruments that are very similar to the referenced stock. Accordingly, assuming a one-time test is needed, we propose the following ratio-based test. We suggest use of a dollar-based test as a backstop to prevent some game-playing, which we discuss further below.

To describe the ratio-based test, we require some notation. Let $S_0\Delta_1$ denote the price of the number of implicit shares at the time of the test, let $P_0$ denote the price of the instrument at the time of the test, and let $r_0$ be the ratio $r_0 = S_0\Delta_1/P_0$. The test measures whether $r_0$ is below a specified threshold, such as 0.7, and if it is, no dividend equivalent payments will be deemed made with respect to the instrument. In the complementary situation, when $r_0$ meets or exceeds the threshold, the instrument will give rise to dividend equivalent payments.

A special rule must be made in situations for which $P_0 < 0$. In those cases, if $S_0\Delta_1 > 0$, the instrument is deemed to give rise to dividend equivalent payments, even though the value of $r_0$ may be negative or undefined, because the instrument represents the use of leverage to maintain a long implicit stock position. Otherwise, if $P_0 < 0$ and $S_0\Delta_1 < 0$, the instrument is deemed not to give rise to dividend equivalent payments, even though the value of $r_0$ may be positive or undefined, because the instrument represents a short implicit stock position.

The ratio $r_0$ is exactly equal to the elasticity of the price of the instrument with respect to stock price. This is defined as the percentage change in the price of the instrument divided by the corresponding percentage change in the price of the stock. This elasticity is thus analogous to the concept of delta used in the proposed regulations, but it avoids the need to specify a particular number of referenced shares because percentage changes are independent of the number of shares — or instruments — involved.

It is interesting to consider the results the new test would produce in Examples 1 and 2, above. The instruments in those examples represented leveraged long implicit stock positions, and so the new test would treat all the instruments as giving rise to dividend equivalent payments. Thus, the differences in treatment between U and T in Example 1 and P and R in Example 2 would cease to exist. More broadly, in any example in which two parties have economically equivalent positions, the new test would prescribe the same result for both.
The new test identifies some instruments as exceeding its threshold, even though the test of the proposed regulations would have reached a different conclusion. For example, consider a call option on a single share of stock that has a delta less than 0.7. The price of the implicit shares will generally exceed the price of the option, that is, $S_0 \Delta_1 > P_0$. As a result, the value of $r_0$ will typically exceed 1, and so the call option would be deemed to give rise to dividend equivalent amounts even though the test of the proposed regulations would reach a different conclusion. This treatment seems appropriate, however, because the call option represents the use of leverage to maintain a long position in stock.

For some other instruments, the new test reaches the conclusion that no dividend equivalent amounts should be deemed made, even though the proposed regulations would have reached the opposite result. For example, consider a call option on a single share of stock, with a delta that exceeds 0.7, and a convertible bond that provides for a single final payment at the expiration of the call, consisting of a principal payment equal to the strike price of the call and interest in an amount equal to the final value of the call. Assume the entire payment is to be made with a single share of stock if the call expires in the money. Under the proposed regulations, the call and the convertible bond have the same number of referenced shares and the same delta, and so the convertible bond gives rise to dividend equivalent amounts, or not, just as the call in isolation does. Under the new test, however, if the strike price is large enough relative to the value of the implicit shares, the ratio $r_0$ for the convertible bond may be less than the specified threshold, and so no dividend equivalent amounts would be deemed made regarding the convertible bond. For a more general example of this type of situation, see Example 3 below.

If some of the differences between the new test and the proposed regulations do not produce the desired policy outcomes, exceptions and special rules can be made to accommodate particular situations. It seems, however, that the new test is the appropriate default test for instruments in general, because it is essentially the same as the test of the proposed regulations, except that it uses percentage changes instead of absolute changes to define the relevant ratio. That is to say, it uses elasticity instead of delta. This change is in some sense minor because elasticity is just delta multiplied by stock price and divided by the price of the instrument. The minor difference is important, however, because it is precisely what enables the new test to avoid any need to use a particular number of referenced shares and to avoid the indeterminacies associated with such a number.

Potential Abuses and Additional Proposals

Elasticity evaluates the true economic nature of a particular instrument at a particular time, but it is still possible to game a one-time elasticity-based test by using additional instruments not subject to testing or engaging in strategies that change elasticity over time. To deal with those concerns, further safeguards may be needed, such as alternative tests, periodic retesting, and antibuse rules. In this section, we present examples illustrating potential problems and describe some ways of addressing them.

Example 3: Suppose an instrument has a high elasticity. An investor may purchase a modified instrument that combines the high-elasticity instrument with a long risk-free bond and thereby reduce the overall elasticity. If the price of the long bond in isolation is $n$ times that of the original instrument, the elasticity of the combined instrument is $1/(n+1)$ times that of the original one. For example, if $n = 1$, the elasticity is cut in half.

The long bond position in the combined instrument of the example may be funded by separate borrowing. The borrowing hedges the long bond, and when viewed together, the borrowing and the combined instrument are essentially the same as the original instrument in isolation. Thus, an investor may transform a high elasticity into a low one and potentially circumvent the intended result of an elasticity-based test, under the assumption that the additional borrowing is not considered when elasticity is determined.

There is a simple alternative test that would fix the problem presented by Example 3 in many situations. Instead of computing the entire elasticity ratio, simply compute the numerator, equal to the price of the implicit shares. If this price is greater than a specified threshold dollar amount, deem the instrument to give rise to dividend equivalent amounts, regardless of what the elasticity test would require. The overall price of the instrument is irrelevant in this case, and so the type of dilution seen in Example 3 is impossible.13

The dollar-based test would likely focus just on very large dollar positions, and so it may not fully capture all appropriate instruments, such as those with high elasticity but more modest dollar values at stake. Accordingly, we suggest that an elasticity-based test be used as the primary identifier of

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13Related instruments and transactions must be aggregated together, because otherwise an instrument could simply be split into pieces small enough to avoid the relevant dollar threshold.
appropriate instruments and that a dollar-based test be used as a backstop that identifies specific additional instruments.

Even a combination of the two tests cannot prevent all problems, as shown by the next example. Consider a “coin-flip” instrument that requires a fair coin to be flipped one day after the instrument is issued. If the coin comes up heads, the instrument becomes a long forward contract, but if it comes up tails, the instrument becomes the complementary short forward contract. The delta of this instrument at issuance is zero, because the deltas of the long and short forward contracts are the negatives of each other, and there is a 50-50 chance that each will occur. Similarly, the number of implicit shares at issuance is zero. Thus, the instrument would not exceed the threshold of either an elasticity-based or a dollar-based test.

**Example 4:** Suppose that an investor buys the coin-flip instrument and holds it for one day. If the coin comes up tails, he closes out the short position and buys a new coin-flip instrument. If it comes up heads, he stops buying new instruments and just holds his long forward. After \( n \) days of doing this, the probability that he has a long forward with respect to the stock is \( 1 - (\frac{1}{2})^n \). For example, after 10 consecutive days, the probability that he holds a long forward is more than 99.9 percent.

The investor in the example easily gets past both a one-time elasticity-based test and a one-time dollar-based test. Nevertheless, he almost certainly obtains a long forward position on the stock in a short period. It seems that the best way to address this sort of variation in elasticity and implicit shares would be to require periodic retesting of the investor’s positions after acquisition, perhaps annually, for example. If retesting is administratively unworkable, rules for combining transactions and antiabuse rules, such as those found in prop. reg. section 1.871-15(l) and (n), may be necessary to deal with this type of situation. Applying antiabuse rules properly may be challenging; however, because it is possible to devise instruments and strategies that achieve results similar to those of the example but do so much less transparently.

**Conclusion**

We have shown with examples that the concept of the number of referenced shares is a problematic feature of prop. reg. section 1.871-15. In the case of defining the dividend equivalent amount, it is easy to eliminate any mention of the number of referenced shares without changing the substance of the proposed regulations, and we suggest that this be done. In the case of the 0.7 delta test, we suggest a revised test that does not use the number of referenced shares and focuses on purely economic features of an instrument, rather than its labeling.

The new test we propose replaces delta with elasticity, essentially replacing absolute changes in instrument and stock prices with percentage changes. This creates a relative perspective that allows for a more robust and accurate measure of the instrument. It also focuses the test on the price of the economically implicit number of shares relative to the price of the instrument.

Assuming a test of the general sort in the proposed regulations is to be implemented, our proposal to use elasticity provides a substantial improvement. Nevertheless, significant concerns remain. The new test still depends on reference to a particular type of stock, although not a particular number of shares, and thus the instrument’s labeling of the particular stock is still critical. Also, it is possible to game the test using either additional instruments not subject to testing or strategies that alter elasticity over time. To address those problems, we propose a backstop dollar-based test that requires dividend-equivalent treatment for instruments with a large dollar value of implicit shares, even though such treatment would not be required under the elasticity-based test. We propose that testing be performed periodically rather than only once. Insofar as this is not administratively feasible, an antiabuse rule would be particularly important to deal with strategies that manipulate elasticity before and after testing.

Finally, the concepts discussed here are important not only for the regulations under section 871(m) but also for any delta-based tests that might be implemented in different contexts, for example, definitions of constructive ownership and constructive sales. It is our hope that the examples and proposals presented here will help in crafting regulations in other areas as well.