Discretionary disclosures using a certifier

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This paper studies two disclosure regimes when a firm with superior private information must rely on a strategic certifier to disclose credibly its prospects. In the ex ante (ex post) disclosure regime, the firm must decide on whether to hire the certifier before (after) observing the certifier’s noisy assessment. Endogenously determined certification fees can actually cause the disclosure probability to decrease in disclosure precision. In the ex ante regime, favorable disclosures are more informative than unfavorable disclosures because of additional positive signaling effect. In the ex post (ex ante) regime, the certifier has incentives to increase (decrease) the disclosure precision.

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1. Introduction

The disconnect between the full disclosure predictions in the early analytical disclosure literature (Grossman, 1981 and Milgrom, 1981) and the empirically observed phenomenon of firms sometimes withholding information inspired the development of voluntary disclosure literature.2 In his seminal papers, Verrecchia (1983, 1990) provides perhaps one of the first explanations for the phenomenon of firms concealing some information: the presence of proprietary disclosure costs can lead firms to disclose only relatively favorable information and withhold unfavorable information in equilibrium.3

This disclosure literature assumes that if firms choose to disclose, they must only disclose their private information truthfully; the only other choice firms have is not to make any disclosures. In contrast, the trading literature (e.g., Akerlof, 1970) often assumes that sellers cannot communicate their private information credibly. While both these sets of assumptions may be true in a variety of settings, we also empirically observe settings in which the credibility of firms’

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2 The unraveling principle (Grossman, 1981 and Milgrom, 1981) predicts that firms will fully disclose their private information under the following circumstances: (1) the firm always obtains private information, (2) investors know that the firm always has the private information, (3) both the firm and investors are Bayesian, rational players and have common priors, (4) the only disclosure options available to the firm are to disclose its private information truthfully or withhold it, but the firm cannot misreport it, (5) disclosing information is costless, and (6) the firm seeks to maximize its expected stock price.
3 Jovanovic (1982) also examines disclosure models in which cost of disclosure can support non-disclosure of unfavorable private information by firms. Alternative explanations for the failure of unraveling in real world include Dye (1985) and Jung and Kwon (1988). While Verrecchia (1983, 1990) emphasize the presence of disclosure costs as an explanation for non-disclosures in equilibrium, Dye (1985) and Jung and Kwon (1988) offer investors’ uncertainty about the manager’s information endowment as another possible explanation for non-disclosures in equilibrium.
disclosures fall in between these two sets of assumptions (Lizzeri, 1999). In particular, we observe firms sometimes relying on independent certifying agencies to make their disclosures credible. In such settings, having firms’ disclosures certified by a neutral entity provides firms a credible means of communicating their private information.

Our analysis extends Verrecchia’s (1983, 1990) by endogenizing the disclosure cost. We model the disclosure cost as the certification fee charged by a monopoly certifier in exchange for its certification service. The firm is incapable of communicating its superior private information about its future prospects in a credible manner and, therefore, must hire the certifier if it wishes to make any credible disclosures to the market. We examine a setting in which the certifier first announces the fee it will charge the firm in return for disclosing the certifier’s noisy assessment of the firm’s future prospects. The firm’s objective is to maximize its expected stock price net of certification costs. Investors set the firm’s stock price as equal to the firm’s expected cash flow conditioned on all publicly available information. In such a setting, we examine and contrast two different disclosure regimes: the ex post and ex ante disclosure regimes. In the ex post (ex ante) disclosure regime, the manager decides whether to hire the certifier after (before) observing the certifier’s noisy assessment of the firm’s prospects.

In the ex post disclosure regime, our analysis predicts that when the owners of the firm enjoy limited liability (and, hence, equity prices are always non-negative), the likelihood of the firm making a disclosure is non-monotonic in the disclosure precision. There are two effects that contribute to this non-monotonic result. First, as Verrecchia (1983, 1990) points out, holding the disclosure cost fixed, the disclosure likelihood increases in the precision of disclosure. When disclosure costs are exogenous, an increase in disclosure precision leads to more skeptical market beliefs in the absence of disclosure, which in turn increases the firm’s disclosure incentives. We refer to this first effect as the direct effect. However, when disclosure costs are endogenous, a second (indirect) effect arises. This second indirect effect that we identify is that, in the ex post disclosure regime, the monopolist certifier seeking to maximize his expected profits increases his certification fees as the disclosure precision goes up.

Our analysis establishes that for low values of precision, the indirect effect dominates the direct effect identified by Verrecchia (1990), causing the disclosure likelihood to decrease in disclosure precision. However, for higher values of precision, the direct effect identified by Verrecchia dominates the indirect effect, thus causing the disclosure likelihood to increase in disclosure precision.

Further, if the owners of the firm were not protected by limited liability, our analysis predicts that the certifier’s expected profit maximizing behavior would lead to the probability of disclosure being independent of its operating risks and the quality of its disclosure. This independence result would follow because the certifier’s optimal fee choice would also vary in disclosure precision. Thus, in the absence of limited liability for the owners of the firm, if the disclosure precision went up (down) the certifier would increase (decrease) his fees in such a manner that the disclosure likelihood would remain unaffected.

These results again contrast with Verrecchia (1983, 1990) who establishes that when the firm is capable of communicating its private information credibly on its own but at a given cost, the likelihood of disclosure increases in the quality of its disclosure.

We next examine the ex ante disclosure regime in which, as explained before, the firm first privately observes a perfect signal about its prospective cash flow, but must decide whether to hire the certifier before observing the noisy assessment that the certifier will obtain (and disclose) if hired by the firm.

In this ex ante disclosure regime, we find that the firm’s disclosure probability actually decreases in disclosure precision. A disclosure conveys two types of information: first, it reveals the noisy assessment of the certifier (the literal effect). Second, a disclosure signals that the firm’s own private information must have been favorable enough to warrant incurring

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1 The role of verification in enhancing credibility of firms’ voluntary disclosures is well recognized in several streams of empirical literature as well. For instance, Ball et al. (2012) document how “independent verification of outcomes disciplines and hence enhances disclosure credibility.” They also find that incurring higher audit fees is associated with more credible, voluntary management forecasts.

Further, Minnis (2011) examine the behavior of private unlisted firms who get their financial statements voluntarily audited, particularly on the eve of borrowing funds. Though auditing is not mandatory for such private firms, Minnis shows that such firms’ disclosures become more informative with auditing and, as a consequence, lenders gain greater confidence in such private firms’ voluntarily audited disclosures.

2 Certification fees are an important source of disclosure cost in capital markets. Butler and Rodgers (2012) analyze a sample of 360 bonds issued during 1997 in the U.S. In their sample, rating agencies’ fees average $157,687 ($150,000 median), which represents about 65 basis points of the principal sum, but can range much higher (90th percentile is $750,000). The average cost is of similar magnitude to the gross spread charged by investment banks for bringing the bonds to market, though medians suggest that underwriters charge greater fees – gross spreads average 83 basis points (median 66 basis points).

3 The list of certifiers in various fields is long and includes auditors and rating agencies in financial services industries, national and international certification agencies such as International Organization for Standardization (issuing certification standards such as ISO 9000, ISO 9000, ISO 14000, ISO 27000, and ISO 22000) and laboratories such as Underwriters Laboratory for manufacturing firms, ISO standard ISO 9126 for software testing, appraisers, engineering and architect firms engaged in certification services, etc.

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5 We use the expressions “the firm makes a disclosure” (“no disclosure”) to mean that “the firm hires the certifier to issue a certified report” (“the firm does not hire the certifier”). Further, the expression “the firm’s disclosure” (and variants of this expression) is used as a short hand to mean that the firm hires the certifier to make a credible disclosure. Also, we use the term “the firm” to refer to the first generation/current owners of the firm.

Finally, we use the term “disclosure cost” to refer to the certifier’s fee that the firm must incur if it wishes to make any credible disclosures.

6 This sequence of events in which the firm must make its disclosure decision (i.e., whether to hire the certifier decision) before observing the certifier’s signal is representative of a variety of settings. For instance, when the firm, with superior private information about its own operating environment, hires a rating agency, the firm is unlikely to be able to predict precisely the assessment that the rating agency will eventually form after the latter’s own analysis. In other words, relative to the firm’s own private information, the actual rating the market observes is noisy.
the cost of hiring the certifier to make a credible disclosure (the positive signaling effect). A lower disclosure precision benefits the marginal firm (namely the one that is indifferent between hiring and not hiring the certifier) because such a firm expects a more favorable market reaction to its disclosure when disclosure precision is lower. This effect arises because, when precision goes down, the market price attaches a greater weight to the signaling effect (and a lower weight to the literal meaning) of the disclosure, and the signaling effect is on average more favorable than the literal meaning effect at the margin. Thus, by increasing the benefits of certification at the margin, a lower disclosure precision results in a higher probability of disclosure.

Further, we find that in the ex ante regime, the positive signaling effect induces a relatively muted market response to unfavorable disclosures because the market perceives less favorable disclosures as relatively noisier than favorable disclosures. As a consequence, our analysis predicts that the equilibrium market response to favorable disclosures is stronger than to less favorable disclosures. It follows that the firm’s stock price is an increasing and convex function of disclosures.

A comparison of the ex ante and ex post disclosure regimes reveals the following. Though the presence of signaling effects in the ex ante regime might seem to suggest that the probability of disclosure would be higher in such a regime, the opposite is true when disclosure precision is very low. The reason is that, in the ex post regime, very low precision leads to very low certification fees whereas in the ex ante regime the certification fee is maximized when precision is lowest. Our analysis also shows that if the certifier could choose the precision of his signal, he would minimize precision in the ex ante regime and maximize it in the ex post regime.

Finally, our analysis also establishes that bad news disclosures are more likely in the ex post disclosure regime than in the ex ante regime. This result follows because the disclosure threshold (in terms of the true firm value) in the ex ante regime is always positive whereas the disclosure threshold (expressed in terms of the noisy certifier’s signal) can be negative in the ex post regime for sufficiently low values of disclosure precision.

This latter result about differences in disclosure threshold values in the two disclosure regimes follows because for low values of disclosure precision, the optimal certification fee is very low (very high) in the ex post (ex ante) regime. As a consequence, for sufficiently low disclosure precision, in the ex post (ex ante) regime, after realizing marginally bad news, the firm’s expected incremental benefits from disclosure over non-disclosure are (not) large enough to cover the low (high) disclosure cost.

Besides Verrecchia (1983, 1990), Lizzeri (1999) is the other most closely related paper to our work. Lizzeri’s focus is on establishing conditions under which the certifier chooses a completely uninformative certifying technology and is still able to extract entire rents from the firm. In contrast, our analysis emphasizes a firm’s proclivity to disclose under different information structures and also when shareholders of an incorporated (and publicly listed) firm enjoy limited liability.

Section 2 introduces the model and distinguishes between the ex post and ex ante disclosure regimes. Section 3 analyzes the ex post disclosure regime, while Section 4 examines the ex ante disclosure regime. Section 5 compares the two disclosure regimes and Section 6 concludes the paper. All proofs are in the Appendix.

2. Model

We adapt the disclosure model studied by Verrecchia (1983, 1990). The firm’s (equity) value is given by

\[ \tilde{y} = \max(\tilde{x}, 0), \]

(1)

where the firm’s cash flow \( \tilde{x} \) is normally distributed with mean zero and variance \( \sigma^2 \), where \( \sigma^2 \) may be viewed as a measure of the firm’s operating risk. The presence of a lower bound for the firm’s value captures the notion that the owners of the firm are protected by limited liability. While the traditional disclosure literature allows for negative equity prices for tractability reasons (and, perhaps, presumably supported by the implicit assumption of unlimited personal wealth for the firm’s owners), we empirically observe that owners of incorporated firms do enjoy limited liability. One natural consequence of this assumption is that the market value of equity must be in all states of the world be weakly greater than zero.

The firm privately observes the realized value of \( \tilde{x} \). Unlike prior disclosure literature (e.g., Grossman, 1981; Milgrom, 1981; and Verrecchia, 1983, 1990), we assume that the firm is unable to communicate its private information \( \tilde{x} \) in a credible manner to the market on its own. Instead, the firm has the option of hiring a certifier to communicate its private information in a credible fashion to the market. If hired, the certifier observes a noisy signal

\[ \tilde{s} = \tilde{x} + \tilde{e} \]

Thus, our predictions are consistent with the asymmetric market response documented by Hayn (1995), who finds the market response to firms’ positive disclosures being more sensitive than negative disclosures.

Given that we do not model any capital structure issues, we use the terms “value” to mean the equity value of the firm.

As will be seen from the following analysis, the assumption of mean zero is without loss of generality.

We use the “tilde” on the top of a variable to denote that it is a random variable (e.g., \( \tilde{x} \)), while the realized value of the random variable is referred to without the tilde (e.g., \( x \)).

Fischer and Verrecchia (1997) also examine a firm’s disclosure strategy in the presence of limited liability, but their focus is on how a firm’s capital structure affects its disclosure policies.

Our assumption that the firm must hire a certifying intermediary to be able to make credible disclosures follows Lizzeri (1999) and reflects the absence of an enforcement mechanism for monitoring and levying sufficiently deterring damages/punishment for any misreporting by the firm. We abstract away from such issues as possible collusion between the firm and the certifier or any other agency problems with respect to the certifier’s behavior. In other words, the credibility of the certifier is not the focus of our analysis here.
and communicates truthfully that signal to the market. The error term \( \tilde{e} \) is normally distributed with mean zero and variance \( \sigma_e^2 \). The precision of the error term \( \tau_e = 1 / \sigma_e^2 \) may also be viewed as a measure of the informativeness (or quality) of the firm’s disclosure. We assume that the distributions of \( \tilde{e} \) and \( \tilde{x} \) are independent. If the certifier is not hired, no disclosure is made.\(^{14}\) Further assume that for exogenous liquidity reasons, the firm’s objective is to maximize the expected price of the firm, \( E[P(\tilde{y})] \). The certifier chooses the certification fee \( c \) to maximize his expected profits.

The time-line of the game follows. At the outset, a certifier with a publicly known disclosure precision, \( \tau_e \), commits to charging a fixed fee \( c \) in return for disclosing \( \tilde{s} \), his noisy assessment of the firm’s cash flow \( \tilde{x} \). If the firm hires the certifier, the latter discloses \( \tilde{s} \). If the firm decides not to hire the certifier, the market observes no credible new information. Conditioned on all the publicly available information, the market prices the firm in a rational and Bayesian-consistent fashion. All players are assumed to be risk neutral and the entire structure of the game is common knowledge.

**Ex post versus ex ante disclosures.** The following analysis contrasts two types of disclosure settings studied in prior literature.\(^{15}\) First, Section 3 examines the setting in which the firm decides whether to hire the certifier (or equivalently, whether to make a disclosure) after observing the realized value of \( \tilde{x} \). In this “ex post” disclosure setting, the firm knows precisely the value of information that will be disclosed to the market if it decides to hire the certifier. Prior literature that examines such ex post disclosure settings include Verrecchia (1983, 1990). Section 4 examines the second type of disclosure setting, referred to as the “ex ante” disclosure setting, where the firm must decide on whether to hire the certifier with the knowledge of only the realized value of \( \tilde{x} \), and before the certifier generates his noisy signal, \( \tilde{s} \). Hence, when hiring the certifier, the firm faces uncertainty about the actual value of the signal \( \tilde{s} \) the market will observe. Prior literature with similar ex ante disclosure settings include Diamond and Verrecchia (1991), Leuz and Verrecchia (2000). However, notice that unlike the prior ex ante disclosure literature, in this paper’s setting, the firm makes the disclosure decision based on its superior private information about its prospective cash flows, \( \tilde{x} \).

### 3. Ex post disclosures

This setting assumes that the firm decides on hiring the certifier after observing the certifier’s signal \( \tilde{s} \). As will be clear from the following analysis, this setting is equivalent to the one in which the manager observes only \( \tilde{x} \) and not the pair \( (\tilde{x}, \tilde{s}) \), because in equilibrium the market observes only the firm’s disclosure (or non-disclosure) of \( \tilde{s} \). Hence, this setting is similar to Verrecchia (1983, 1990), except for the endogeneity of the disclosure cost and the limited liability assumption.

Following Verrecchia (1983), for a given certification fee \( c \), we conjecture the existence of a unique disclosure threshold \( k \) that is both optimal for the firm and consistent with market’s beliefs. Under such a threshold strategy, the firm hires the certifier if and only if \( \tilde{s} \geq k \), and the threshold level conjectured by the market \( \tilde{k} \) is equal to the actual threshold \( k \) used by the firm in equilibrium. Hence, the equilibrium price given nondisclosure, \( P(\text{ND}) \), is consistently set as

\[
P(\text{ND}) = E[\tilde{y}|\tilde{s} \leq k].
\]

Since price given disclosure of \( \tilde{s} \) is given by

\[
P(s) = E[\tilde{y}|\tilde{s}],
\]

it follows that the value of the threshold \( k \) is defined by

\[
\begin{align*}
E[\tilde{y}|\tilde{s} = k] - c &= E[\tilde{y}|\tilde{s} \leq k].
\end{align*}
\]

The first term on the left hand side of the above equation represents the expected price given disclosure of \( \tilde{s} = k \). The right hand side of the above equation represents the firm’s price given nondisclosure. Given the disclosure cost \( c \) and the realized value of \( \tilde{s} = k \), Eq. (4) implies that the firm is indifferent between disclosing and not disclosing given \( \tilde{s} = k \).

Given a certification fee \( c \), we can express the certifier’s expected profits as

\[
\Pi = c \times \Pr(\tilde{s} \geq k(c)),
\]

where \( \Pr(\tilde{s} \geq k(c)) \) is the probability that the firm hires the certifier for any given fee \( c \).\(^{16}\)

For future reference, we define the function

\[
\hat{\Delta}(k) \equiv E[\tilde{y}|\tilde{s} = k] - E[\tilde{y}|\tilde{s} \leq k],
\]

which represents the relative benefit of disclosure at the margin, when the market correctly conjectures that the firm uses the threshold level \( k \). The indifference condition (4) implies that

\[
\hat{\Delta}(k) = c.
\]

We are now in a position to define an equilibrium for this game. Let the firm’s disclosure (certifier-hiring) strategy be defined by \( d = d(s) \in \{s, \text{ND}\} \), where ND denotes no disclosure.

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\(^{14}\) As noted in the Introduction, we adopt the convention that when we say “the firm makes a disclosure (no disclosure)” (or any variation of this statement), we mean that “the firm hires the certifier to issue a certified report (the firm does not hire the certifier).”

\(^{15}\) We would like to thank an anonymous referee for suggesting that we frame our analysis in these terms.

\(^{16}\) To reduce notational clutter, hereafter we simply use the notation \( k = k(c) \). That is, the certifier must anticipate the likelihood of the firm hiring his services when he determines his fees \( c \).
Definition of an equilibrium. An equilibrium in this game consists of the certifier determining a fee $c^*$, a disclosure strategy by the firm, $d = d(s)$, and a market price $P(d(s))$ such that

(a) the certifier’s choice of the fee $c^*$ maximizes its expected profits as given in (5);
(b) given the realized value of the signal, $\delta$, the firm decides on hiring the certifier to maximize the expected market price $P = P(d(\delta))$; and
(c) the equity market correctly conjectures the disclosure strategy used by the firm in equilibrium and determines the price of the firm as the expected value of $\bar{y}$ conditioned on all the information available to the market.

As a first step in establishing the existence of a unique equilibrium in this game, the following lemma characterizes the function $\Delta(k)$.

Lemma 1. (a) The function $\Delta(k)$ is non-negative, strictly increasing in $k$, $\lim_{k \to -\infty} \Delta(k) = 0$ and $\lim_{k \to \infty} \Delta(k) = \infty$. Hence, for any given fee $c > 0$, the disclosure sub-game between the firm and the market has a unique equilibrium.

The above lemma establishes that for any given certification fee $c$, Eq. (7) is solved by a unique threshold level $k$. In this way, Lemma 1 together with Eq. (7) establishes a monotonic relation between the certification threshold $k$ and the certification fee $c$. Hence, we can think of the certifier as effectively choosing the probability of certification by his choice of the fee $c$. Let $\Phi(\cdot)$ and $\phi(\cdot)$ denote the cumulative distribution function and probability density function of the standard normal distribution, respectively. Combining Eqs. (7) and (5), the certifier’s expected profits are given by

$$\Pi = c \times \Pr(\delta \geq k) = \left( E[\bar{y}|\delta = k] - E[\bar{y}|\delta \leq k] \right) \times \Pr(\delta \geq k).$$

(8)

Letting $\rho \equiv \sigma_c / \sigma_s$ and the normalized threshold $z = k / \sigma_s$, one can derive the function

$$\Delta(z) = \frac{\sigma_c^2}{\sigma_s^2} \int_{-\infty}^{z} \frac{\Phi\left(\frac{\tilde{\delta}}{\sigma_s}\right) \Phi(t)}{\Phi(z)} \, dt.$$ 

(9)

Using the expression (20), the expected profits function (8) reduces to

$$\Pi(z) = \Phi(-z) \Delta(z).$$

(10)

As mentioned previously, from an analytical standpoint, one can view the certifier as effectively choosing $z$ to maximize his expected profits $\Pi(z)$.

The following proposition predicts the firm’s disclosure behavior when the certifier’s expected profit maximizing fee determines the firm’s cost of disclosure.

Proposition 1. In this ex post disclosure setting, there exists a unique equilibrium in which:

(i) the equilibrium disclosure threshold is given by

$$k^* = \sigma_c z^*,$$

where

$$z^* = \arg \max_z \Pi(z);$$

(11)

(ii) the optimal certification fee is given by

$$c^* = \Delta(z^*),$$

and

(iii) the price function is given by

$$P(d(s)) = \begin{cases} 
E[\bar{y}|\delta \leq k^*] & \text{if } d = \text{ND}, \text{ and} \\
\sigma \varphi\left(\frac{\tilde{\delta}}{\sigma_s}\right) & \text{if } d = s,
\end{cases}$$

where

$$\varphi(t) = t \Phi(t) + \phi(t)$$

is an increasing and convex function, bounded from below by zero, $\beta = \sigma_c^2 / \sigma_s^2$ and $\sigma = \sqrt{\text{Var}(\tilde{\delta})}$.

Three noteworthy aspects of this proposition are as follows: the first aspect deals with the uniqueness of the equilibrium: (i) the certifier’s optimal choice of his expected-profit-maximizing fee, $c^*$, is unique; and (ii) the equilibrium disclosure threshold choice of the firm, $k^*$, constitutes a unique and optimal response by the firm to the certifier’s choice of fee, $c^*$.  

The second aspect of the proposition is that the probability of disclosure, \( \Phi(z) \), depends both on the firm’s operating risk and the certifier’s disclosure quality \( \rho \). The following corollary elaborates on the firm’s equilibrium disclosure behavior.

The third implication of Proposition 1 is the convexity of the firm’s price in the disclosure \( d \), which means that in equilibrium the market is relatively more sensitive to good news. This convexity arises here because lower signals have relatively weaker impact on the firm value, given the presence of limited liability.

Corollary 1 generates predictions about the probability of disclosure in general, and about the probability of bad news disclosures in particular. Disclosure of bad news is said to occur when the market’s posterior expectation about the firm’s cash flow, \( \tilde{x} \), given the firm’s noisy disclosure \( \tilde{s} = s \) is less than the unconditional prior mean of the firm’s cash flow, \( E[\tilde{x}] \).

Corollary 1. In the equilibrium described in Proposition 1:

(i) the effect of disclosure precision \( \tau_e \) on the probability of disclosure is non-monotonic; and
(ii) further, for any fixed \( \sigma_x \):

\[
\lim_{\tau_e \to 0} \Pr(\tilde{s} \geq k^*) = 0.241, \quad \text{and} \quad \lim_{\tau_e \to 0} \Pr(\tilde{s} \geq k^*) = 0.765.
\]

Verrecchia (1990) demonstrated that for a given cost of disclosure, the probability of disclosure is monotonically increasing in disclosure quality \( \tau_e \). However, this paper finds that when the cost of disclosure is endogenized, this relation does not hold, even in the absence of limited liability. In fact, it can be shown that if equity holders were not to enjoy limited liability, the probability of disclosure would be independent of disclosure quality, \( \tau_e \). Such independence result follows because the expected profit maximizing certifier would change his fee \( c \) in response to a change in disclosure quality \( \tau_e \) in such a manner that the equilibrium probability of him being hired by the firm would not change.

However, when equity holders do enjoy limited liability as in the setting examined here, Corollary 1 demonstrates that the probability of disclosure depends on disclosure quality, \( \tau_e \), but in a non-monotonic fashion. Fig. 1 illustrates, among other results, the intuition for the claim in part (i) of Corollary 1.

While for very high values of disclosure quality \( \tau_e \) (see the top panel in Fig. 1) the probability of disclosure does increase in disclosure quality \( \tau_e \) as in Verrecchia (1990), it also decreases in disclosure quality \( \tau_e \) for low values of \( \tau_e \). Indeed, the disclosure probability decreases steeply in \( \tau_e \), particularly for very low values of \( \tau_e \).

\footnote{For the sake of brevity, we do not include the details of the proof for this claim here. However, the full proof is available from the authors.}
Changes in disclosure quality \( \tau_e \) have two effects on the probability of disclosure. The first direct effect was documented by Verrecchia (1990): for a given fee, a higher information quality leads to a higher probability of disclosure because the market tends to be more skeptical about the value of non-disclosing firms when the precision of the available signal is higher. This naturally increases the firm’s disclosure incentives. The second indirect effect arises here via endogenous changes in certification fee: the optimal certification fee is increasing in disclosure quality. These two effects (direct and indirect) move in opposite directions with the indirect effect dominating the direct effect at lower levels of disclosure quality, leading to a decreasing disclosure likelihood for very low values of \( \tau_e \) (see the top panel in Fig. 1).

As disclosure precision decreases, the market penalizes the firm less for not disclosing the signal. Such lower market penalty occurs because for any given certification fee \( c \), the price benefit from disclosing the signal, relative to not disclosing it, shrinks. Anticipating this effect, the certifier drops his fee sharply when the precision \( \tau_e \) of the signal goes down (see the top panel in Fig. 1). This effect is particularly pronounced under limited liability, because the price is bounded from below by zero, hence the penalties for non-disclosure are limited. This explains why for sufficiently low values of disclosure quality \( \tau_e \), the indirect effect studied here – of the certifier’s equilibrium fee choice changing in anticipation of the shift in the firm’s demand for its services – dominates the direct effect, leading to a negative relation between the probability of disclosure and disclosure quality, \( \tau_e \).

Observe that in this model there is no unraveling: as \( \tau_e \to 0 \) the certification fee will approach zero but the probability of disclosure will still be strictly positive, but less than one. This result contrasts with Lizzeri (1999) partly because of our ex post disclosure setting.\(^{18}\)

Corollary 1 results also have implications for bad news disclosures by firms. Observe that in the equilibrium described in Proposition 1, as characterized by Corollary 1(ii), bad news are released with positive probability only when information quality is sufficiently low. For example, when information quality is zero, the probability of bad news disclosures is close to 26.5%. In contrast, when information quality \( \tau_e \) is sufficiently high, only good news would be disclosed in equilibrium (i.e., \( z^* \geq 0 \)).\(^{19}\)

The final result of this section studies the effect of information quality on the certifier’s profits.

**Corollary 2.** In the equilibrium described in Proposition 1:

(i) The certifier’s expected profits, \( \Pi(z^*) \), increase in information quality \( \tau_e \) and operating risk \( \sigma_x \).

(ii) \[ \lim_{\tau_e \to \infty} \Pi(z^*) = 0.142 \times \sigma_x, \quad \text{and} \quad \lim_{\tau_e \to 0} \Pi(z^*) = 0. \]

The above results suggest that if the certifier could choose the level of information quality without incurring any additional costs to improving the disclosure precision, then he would prefer to choose the highest possible level. Moreover, for any given disclosure precision \( \tau_e \), the certifier’s expected profits increase in the operating risk, \( \sigma_x \), of the firm. This result is also intuitive because with greater operating risk, \( \sigma_x \), lower cash flow realizations of \( \hat{x} \) become more likely in ex ante terms. As a result, for larger values of operating risk, \( \sigma_x \), the market penalizes the firm more for non-disclosures and, hence, the firm has greater incentives to hire the certifier than with lower operating risk.

4. **ex ante disclosure**

The ex post disclosure setting analyzed above assumes that the firm decides on hiring the certifier after observing the realized value of the certifier’s signal \( \hat{s} \). However, sometimes it is natural to think that the firm will have to decide on whether to hire the certifier before observing the final realized value of the certifier’s assessment of the firm’s prospects. This section examines such an ex ante disclosure setting.

As before, we assume that the firm privately observes a perfect signal about the prospective value of cash flow \( \hat{x} \) before it decides whether to hire the certifier. After being hired, the certifier generates the noisy signal \( \tilde{s} = \hat{x} + \varepsilon \). This sequence of events is designed to operationalize the notion that while an external, uninterested, and neutral certifier is more likely to lend greater credibility to the firm’s disclosure, the external certifier is also likely to have less precise information about the firm’s prospects than what the firm itself possesses. Further, the firm must decide on whether to hire the certifier after privately observing the realized value of \( \hat{x} \), but before knowing the value of the noisy signal \( \tilde{s} \) that the certifier would eventually generate. Therefore, hiring the certifier imposes some uncertainty on the firm because for any given value of the firm’s privately realized cash flow signal \( \hat{x} \), the firm can never be certain about the eventual realization of the certifier’s signal \( \tilde{s} \).

\(^{18}\) In Lizzeri (1999), the decision to hire the certifier is based on the firm’s more precise information and, hence, has signaling content. Therefore, Lizzeri’s setting is closer to the ex ante disclosure regime that we analyze next than this ex post regime.

\(^{19}\) If the firm’s owners were not to enjoy limited liability, then regardless of the value of disclosure precision \( \tau_e \), the unconditional probability of disclosure would be approximately 76.5%, with the result that the unconditional likelihood of bad news disclosure would be 26.5%. One can also show that the likelihood of disclosure conditional on the firm observing the bad news is approximately 54% in the absence of limited liability.
The sequence of events in this setting follows: the certifier commits to a fee, \( c \); the firm privately observes the realized true value \( \hat{x} \) and decides whether to hire the certifier; the certifier generates the noisy signal \( \tilde{s} \) only if hired; and the market price \( P(\cdot) \) forms in a Bayesian rational manner.

The equilibrium definition from the previous section can be readily extended to accommodate this ex ante disclosure setting. Assume that the firm uses the disclosure threshold \( k_a \) such that the firm hires the certifier if and only if \( \hat{x} \geq k_a \). Further assume that the market conjectures the disclosure threshold level used by the firm as \( \hat{k}_a \). Since the market’s conjectured threshold would be equal to the actual threshold used by the firm in equilibrium, i.e., \( k_a = \hat{k}_a \), the market price given the firm’s disclosure \( \tilde{s} = s \) is given by

\[
P(s, k_a) = E_s[\tilde{y} | \hat{x} \geq k_a, \tilde{s} = s].
\]  

Note that the price \( P(s, k_a) \) depends not only on the actual disclosure \( \tilde{s} = s \), but also on the market’s posterior belief that the firm must have realized private information \( \hat{x} \) above the conjectured threshold level \( \hat{k}_a \) to trigger the disclosure decision. Note that, as observed above, Eq. (12) invokes the condition that, in equilibrium, the threshold level as conjectured by the market, \( s = \hat{s} \), the actual threshold used by the firm. The firm obtaining superior private information relative to the certifier introduces an additional signaling effect beyond the information contained in the certifier’s disclosure of \( \tilde{s} \) because, in equilibrium, the very decision by the firm to hire the certifier signifies a relatively positive firm’s private information \( \hat{x} \). Thus, the information content of what markets observe from the firm’s noisy disclosure \( \tilde{s} \) is more than the simple literal meaning of the firm’s disclosure \( \tilde{s} \). It is also important to reiterate that such a positive signaling effect of the firm’s disclosure occurs only when the firm decides on whether to hire the certifier before observing the certifier’s signal \( \tilde{s} \). In this way, firms possessing less information about the certifier’s signal actually increases the information content of the firm’s disclosure \( \tilde{s} \) beyond just the literal meaning of its disclosure.

Now, consider the equilibrium in the disclosure sub-game between the firm and the market, for a given disclosure cost \( c_n \). The disclosure threshold \( k_a \) must, in equilibrium, satisfy

\[
E_s[P(\tilde{s}, k_a)|\hat{x} = k_a] - c_n = E_s[\tilde{y} | \hat{x} \leq k_a].
\]  

The above indirectness condition exploits the fact that in equilibrium the market’s conjecture about the threshold \( \hat{k}_a \) must match the actual disclosure threshold \( k_a \) used by the firm. The first term on the left hand side of (13) reflects the expected price from hiring the certifier given \( \hat{x} = k_a \), because at the time of hiring the certifier, the firm is not sure about what message \( \tilde{s} \) would eventually be generated by the certifier. The right hand side of (13) refers to the price given no disclosure. Given \( \hat{x} = k_a \), the firm is indifferent between disclosing \( \tilde{s} \) (which would involve incurring disclosure cost \( c_n \)) and non-disclosure. From condition (13), the relative benefit of disclosure at the margin, given that the market correctly conjectures in equilibrium the actual disclosure threshold \( k_a \) used by the firm, can be written as

\[
\Delta_d(k_a) = E_s[P(\tilde{s}, k_a)|\hat{x} = k_a] - E_s[\tilde{y} | \hat{x} \leq k_a].
\]  

In equilibrium, it must be true that

\[
\Delta_d(k_a) = c_n.
\]  

In this setting, bad disclosures of \( \tilde{s} \) are perceived to be relatively noisier than good disclosures of \( \tilde{s} \) because the information content of bad disclosures is partially offset by the favorable signaling effect attached to the fact that the firm chose to hire the certifier. Since the firm decides on hiring based only on its private information \( \hat{x} \), and the expected value of the certifier’s unbiased signal is equal to the realized value of \( \hat{x} \), the very decision by the firm to hire the certifier signifies to the market that the firm’s private information \( \hat{x} \) must be reasonably good even when the certifier’s ex post disclosure \( \tilde{s} \) turns out to be relatively bad. Thus, for any given disclosure cost \( c_n \), the very fact that the firm chose to hire the certifier prompts the market to be ‘more generous’ to the firm for low values of the certifier’s disclosure, \( \tilde{s} \), than in the ex post setting. Anticipating this relatively generous response of the market, one would naturally expect that for any fixed certifier’s fee \( c_n > 0 \), the firm would “overinvest” in its disclosure decision in this ex ante disclosure setting relative to the ex post disclosure setting.

The next lemma establishes the convexity of the firm’s price function, following its disclosure of the noisy signal \( \tilde{s} \).

**Lemma 2.** The price function \( P(s, k_a) \) is an increasing and convex function of the certifier’s disclosure \( s \), and is bounded from below by \( \max(0, k_a) \).

The convexity of the price in the certifier’s disclosure \( s \) arises here because the informativeness of the certifier’s signal increases in the magnitude of \( s \). As noted above, less favorable disclosures of \( \tilde{s} \) are perceived to be relatively noisier because of the positive signaling effect attached to the firm’s decision to hire the certifier. Since the firm decides on hiring based only

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\[ \text{Note:} \quad \text{The subscript 'a' in } k_a \text{ and other notations below (e.g., } \Pi_n, c_n, \text{ and } \Delta_d \text{) refer to this ex ante disclosure setting to distinguish from the disclosure threshold } k (\Pi, c, \text{ and } \Delta) \text{ used in the prior setting where the firm decided on disclosure ex post observing the realized value of the certifier's signal } \tilde{s}. \]

\[ \text{Further note that the threshold } k_a \text{ in this ex ante setting is defined in terms of the firm's cash flow, } \hat{x}, \text{ whereas the threshold } k \text{ in the ex post setting discussed above is defined in terms of the certifier's noisy signal } \tilde{s}. \]

\[ \text{Recollect from the previous section that if the firm decided on whether to hire the certifier after observing the certifier's signal } \tilde{s}, \text{ then the firm's disclosure decision would depend only on the value of the noisy signal } \tilde{s} \text{ it observed and would be independent of its own superior private information } \hat{x}. \]
on its private information $\bar{x}$ and before the certifier generates his signal $\bar{s}$, any favorable disclosures of $\bar{s}$ tend to be viewed as less noisy than unfavorable disclosures of $\bar{s}$. Thus, the ex ante disclosure decision by the firm contributes significantly to the convexity of the pricing function.

Before proceeding with further analysis, we make the following approximation assumption to simplify the proofs:

**Assumption 2.**

$$E_\bar{s}[P(\bar{s}, k_a) | \bar{x} = k_a] \approx P(E_\bar{s}[\bar{s} | \bar{x} = k_a], k_a).$$  \hfill (16)

The approximation in (16) is obtained by taking a first-order Taylor expansion of the price function around the mean of the certifier’s signal $\bar{s}$ conditional on $\bar{x} = k_a$. This approximation is very accurate when the certifier’s information technology is noisy (in fact, in the limit as $\sigma \to \infty$, the approximation is perfect). Further, the subsequent results do not depend on this approximation in a qualitative manner. $^{22}$

Using the above approximation, the next Lemma establishes the expected market price for the firm given that $\bar{x} = k_a$.

**Lemma 3.**

$$E_\bar{s}[P(\bar{s}, k_a) | \bar{x} = k_a] = \begin{cases} \frac{\beta k_a \Phi \left( \frac{\beta k_a}{\sigma} \right) + \sigma \phi \left( \frac{\beta k_a}{\sigma} \right)}{1 - \Phi(k_a \sigma)} & \text{if } k_a < 0, \\ \beta k_a + \sigma \delta(k_a \sigma) & \text{if } k_a \geq 0, \end{cases}$$

where $\beta = \sigma_s^2 / \sigma^2$, $\delta(t) = \phi(t)/(1 - \Phi(t))$, and $\sigma = \sqrt{\text{Var}(\tilde{x} | \tilde{s})}$.

As before, we first study the disclosure sub-game for a given certification fee $c_a$. As in the ex post setting, we first establish that there is a monotonic relation between the disclosure threshold $k_a$ and $c_a$. The function $\Delta_a(k_a)$ in (14) is increasing in $k_a$, and $\lim_{k_a \to -\infty} \Delta_a(k_a) = 0$ and $\lim_{k_a \to \infty} \Delta_a(k_a) = \infty$. These properties ensure the existence of a unique equilibrium in the disclosure sub-game for any given fee $c_a$, and enable us to think of the certifier as effectively choosing the disclosure threshold $k_a$ through its choice of the certification fee $c_a$. Holding the disclosure cost $c_a$ fixed, the next Proposition establishes the impact of disclosure precision on a firm’s proclivity to make disclosures in this ex ante disclosure setting.

**Proposition 2.** When the firm must decide on whether to disclose before observing the certifier’s signal $\bar{s}$, for any given certification fee $c_a > 0$, there exists a unique threshold equilibrium and the probability of disclosure decreases in disclosure precision $\tau_s$.

The above proposition establishes that in this ex ante disclosure setting, the likelihood of disclosure by the firm decreases as the certifier’s signal becomes more precise. Though perhaps surprising, this result is explained by the fact that at the margin (i.e., when $\bar{x} = k_a$) the benefit of disclosure increases when the certifier’s signal is noisier: while on average the marginal firm expects to receive a higher price given certification with a noisier signal $\bar{s}$, the price given no disclosure remains unaltered. Put differently, when the certifier’s signal becomes noisier lower firm types are subsidized more by higher types, given certification. So, in contrast to the ex post disclosure setting, for a given disclosure cost here the probability of disclosure always decreases in the disclosure precision $\tau_s$.

While the above proposition examined the firm’s disclosure likelihood when the disclosure cost was given as exogenous, the following analysis establishes, among others, a similar result even when the disclosure cost $c_a$ is endogenously determined.

Next, consider the certifier’s maximizing behavior. The certifier solves the following program:

$$\Pi^*_a \equiv \max_{k_a} \Pi_a(k_a) = \Phi \left( \frac{k_a}{\sigma_\tilde{x}} \right) \Delta_a(k_a).$$  \hfill (17)

The above maximization program implies that the certifier maximizes expected profits, computed as the probability of being hired multiplied by the fee $c_a$, which in equilibrium equals $\Delta_a(k_a)$. Letting $k^*_a$ denote the equilibrium disclosure threshold level used by the firm in this ex ante disclosure setting, the next proposition predicts how the certifier’s maximization behavior in (17) and disclosure quality affect the firm’s disclosure likelihood.

**Proposition 3.** When the firm must decide on whether to disclose before observing the certifier’s signal $\bar{s}$, there exists a unique threshold equilibrium. Further, in the unique equilibrium:

(i) the optimal certification fee is given by

$$c^*_a = \Delta_a(k^*_a),$$  \hfill (18)
where \( k^*_a = \arg \max_{k_a} \Pi_o(k_a) \). In the limit,\(^{23}\)
\[
\lim_{\sigma \to \infty} c^*_a = \sigma_o \delta(0);
\]
(ii) for any finite \( \tau_e \), the probability of disclosure is smaller than 50 \%, i.e., \( k^*_a > 0 \);
(iii) the probability of disclosure decreases in \( \tau_e \) and converges to 50\% as \( \tau_e \to 0 \);
(iv) the certifier’s expected profits decrease in \( \tau_e \); and
(v) finally,
\[
\lim_{\tau_e \to 0} k^*_a = 0;
\]
and
\[
\lim_{\tau_e \to 0} \Pi^*_a = \sigma_o \frac{\delta(0)}{2}.
\]

The above proposition establishes that the unique equilibrium disclosure threshold \( k^*_a \) (as induced by the fee \( c^*_a \) that maximizes the certifier’s expected profits, \( \Pi^*_a \)) is greater than zero, bounded from above, and increasing in \( \tau_e \). So, in this ex ante disclosure setting, for any finite \( \tau_e \), the firm will not hire the services of the certifier after receiving any bad news, i.e., for any \( 0 < \tilde{x} < 0 \). In fact, the firm will not hire the certifier after receiving mildly favorable news, i.e., for \( \tilde{x} \in (0, k^*_a) \). In contrast, recollect that in the ex post setting, the optimal disclosure threshold \( k^* \) (defined in terms of the signal \( \tilde{x} \)) could be negative for sufficiently low \( \tau_e \), implying that the firm will sometimes initiate disclosure even when it fully anticipates bad news disclosure (i.e., \( \tilde{x} < 0 \)), provided that the certifier’s information quality is low enough.

**Proposition 3** has at least one major implication. If the certifier could choose the disclosure quality \( \tau_e \), it would have an incentive to choose low disclosure quality. This result contrasts with the corresponding incentives of the certifier to choose more informative disclosure system in the ex post disclosure setting examined in the previous section. In the ex post disclosure setting studied in Section 3, the certifier has strictly positive incentives to improve the precision of its disclosures because, ceteris paribus, more precise disclosures induce stronger market reactions, thereby allowing the certifier to charge higher fees. In contrast, in the current ex ante disclosure setting, the certifier always prefers to make its disclosures less informative. By increasing the noise of its disclosure, the certifier is able to increase the demand for its services because even firms holding moderately unfavorable private information would have greater incentives to hire the certifier if the certifier’s disclosure is noisy enough because (a) the chances of the certifier’s disclosures being favorable are relatively high, and (b) the firm still benefits from the positive signaling effect attached to the decision of hiring the certifier even if the certifier’s disclosure is unfavorable.

### 5. ex ante versus ex post

In this section, we compare disclosure proclivities of the firm under the two disclosure regimes studied above.

**Proposition 4.** There exists a \( \tau_e \) such that if \( \tau_e \leq \tau_e \), then the probability of disclosure under the ex post regime is greater than the probability of disclosure under the ex ante regime.

**Fig. 2** provides a numerical illustration of Proposition 4 result that for sufficiently low values of disclosure precision \( \tau_e \), the likelihood of disclosure under the ex post disclosure regime is greater than under the ex ante disclosure regime.

How can the probability of disclosure be lower under the ex ante regime given the signaling incentives of that regime? The reason is that, when \( \tau_e \) is small, the optimal fee is significantly greater under the ex ante regime than in the ex post setting (compare the behavior of equilibrium fee choices under the two regimes in Fig. 1). Indeed, in the ex ante regime, a lower \( \tau_e \) increases the willingness to pay for the marginal firm and a higher \( \tau_e \) decreases the likelihood that the marginal firm obtains a better certifier’s signal. This explains why the fee goes down in \( \tau_e \) in the ex ante regime. In contrast, a low \( \tau_e \) in the ex post regime leads to a weak market reaction to the certifier’s disclosure and, as a consequence, a lower willingness to incur the disclosure cost. This produces low certification fees, and a relatively higher probability of disclosure in equilibrium.

Finally, a comparison of the ex ante and ex post disclosure regimes establishes that bad news disclosures are more likely in the ex post disclosure regime than in the ex ante regime. This result follows because for sufficiently low values of disclosure precision, in this ex ante regime the threshold \( k^*_a > 0 \), whereas in the ex post regime, the threshold \( k^* \) can be less than zero.\(^{24}\) These differences in equilibrium thresholds occur because for very low values of disclosure precision \( \tau_e \), the

\(^{23}\) Note that if \( \sigma = 0 \), then this ex ante setting essentially reduces to the ex post setting because the firm would be able to precisely infer the value of \( \tilde{x} \) from its private information about \( \hat{x} \). Hence, such limiting results are not included in this proposition.

\(^{24}\) Recollect that in the ex ante regime, the threshold \( k^*_a \) is expressed in terms of the true value of the firm, \( \hat{x} \), whereas in the ex post disclosure regime, the threshold \( k^* \) is expressed in terms of the certifier’s noisy signal, \( \tilde{x} \).
optimal certification fee is much higher in the ex ante regime than in the ex post regime (compare the right hand side of the top and bottom panels in Fig. 1). As a consequence, for sufficiently low disclosure precision, in the ex post (ex ante) regime, after realizing marginally bad news, the firm’s expected incremental benefit from disclosure over non-disclosure is (not) large enough to cover the low (high) disclosure cost.

Lizzeri (1999) focuses on characterizing the optimal precision choices of the certifier in a setting that is similar to our ex ante setting. Our ex ante setting converges to Lizzeri (1999) when precision goes to zero.

6. Conclusion

This paper examines discretionary disclosures by a firm (whose owners enjoy limited liability) when it must rely on a strategic certifier to make a credible disclosure to the capital market. In the ex post disclosure setting in which the firm decides on hiring the certifier after observing the certifier’s noisy signal, we find that the likelihood of disclosure by the firm is non-monotonic in disclosure precision. However, in the ex ante disclosure setting when the firm decides on hiring the certifier before observing the certifier’s noisy assessment, this paper predicts that (a) the probability of disclosure can actually decrease in the disclosure precision; (b) the probability of bad news disclosure is less than in the ex post disclosure regime; (c) the market response to favorable versus unfavorable disclosures can be asymmetric because the firm’s disclosures contain additional information (about relatively favorable private information of the firm) beyond the literal meaning of the certifier’s disclosure; and (d) the market’s asymmetric response to favorable versus unfavorable disclosures can produce convexity of stock prices in firms’ disclosures. Thus, ceteris paribus, the firm knowing less about the certifier’s signal can actually increase the information content of its disclosures.

Future research could consider oligopolistic certification. Competition among certifiers can potentially produce an opportunity for firms to engage in “shopping for certifiers’ opinions.” If the firm is able to observe or predict each certifier’s prospective signal, then the firm is likely to choose that certifier who is more likely to generate a more favorable report on behalf of the firm (e.g., the alleged practice of some firms engaging in opinion shopping for bond ratings.) In such a setting, each one of the certifiers would then anticipate this possible opinion shopping by the firm for their disclosures and such anticipation would significantly affect their fee choices, which in turn would also influence firms’ discretionary disclosure behavior.

Appendix A

Proofs for Lemma 1 and Proposition 1. 25 We first state and prove a series of preliminary results required to prove Lemma 1 and Proposition 1. It is useful to define the functions $\delta(t)$ and $\lambda(t)$ as follows:

$$\delta(t) = \frac{\phi(t)}{\theta(-t)} - \lambda(-t).$$

25 For the sake of brevity, this Appendix contains only essential parts of proofs. Full details of all proofs are available from authors.
Lemma 4. The value of $\Delta(z)$ can be computed as

$$
\Delta(z) = \frac{\sigma^2_z}{\sigma_z} \left( \int_{-\infty}^{z} \Phi \left( \frac{z}{\rho} \right) \phi(t) \frac{dt}{\Phi(z)} \right).
$$

Proof. By definition

$$
E[y|s=k] = \int_{0}^{\infty} y \phi(x|s=k) \, dx.
$$

Applying some standard results, the above expression reduces to

$$
E \left[ y | s = k \right] = \sigma \phi \left( \frac{bk}{\sigma} \right),
$$

where $\phi(x) = x \phi(x) + \phi(x)$, with $\phi(x) = \phi(x)$; $\beta = \frac{\sigma^2_z}{\sigma^2_s}$ and $\sigma = \sqrt{\text{Var}(x|s)} = \sigma_s \sigma_s$. After some algebra, we arrive at

$$
\Delta = \sigma \left( \int_{-\infty}^{z} \phi \left( \frac{bk}{\sigma} \right) - \phi \left( \frac{bz}{\sigma} \right) \right) = \sigma \int_{-\infty}^{z} \beta \phi \left( \frac{bz}{\sigma} \right) \phi \left( \frac{s}{\sigma} \right) \, dz.
$$

Letting $t = s/\sigma_z$ and $z = k/\sigma_z$, and changing variables leads to

$$
\Delta(z) = \sigma \int_{-\infty}^{z} \beta \phi \left( \frac{dz}{\sigma} \right) \phi \left( \frac{s}{\sigma} \right) \, dz = \sigma \int_{-\infty}^{z} \beta \phi \left( \frac{s}{\sigma} \right) \, \Phi(t) \, dt.
$$

Proof of Lemma 1. First, inspection of Eq. (20) reveals that $\Delta(z)$ is strictly positive for all $z$. We further note that $\lim_{z \to -\infty} \Delta(z) = 0$ and $\lim_{z \to \infty} \Delta(z) = \infty$. By continuity, this demonstrates the existence of an equilibrium for any $c > 0$. Second, we establish that $\Delta(z)$ is a strictly increasing function of $z$, which ensures the uniqueness of the equilibrium for any given certification fee. Differentiating $\Delta(z)$ yields

$$
\frac{\partial \Delta(z)}{\partial z} = \phi \left( \frac{z}{\rho} \right) - \phi(z) \left( \int_{-\infty}^{z} \phi \left( \frac{z}{\rho} \right) \frac{dt}{\Phi(z)} - \int_{-\infty}^{z} \phi \left( \frac{z}{\rho} \right) \phi(t) \frac{dt}{\Phi(z)} \right),
$$

where the last inequality follows from some simple algebra and applying the standard result that $\lambda'(x) \leq 0$ (Heckman and Honore, 1990). Integration by parts shows that

$$
\int_{-\infty}^{z} \phi \left( \frac{z}{\rho} \right) \phi(t) \, dt = \Phi \left( \frac{z}{\rho} \right) \phi(z) - \int_{-\infty}^{z} \phi \left( \frac{z}{\rho} \right) \phi(t) \, dt.
$$

Hence, plugging this back into (21) and after some algebraic reduction we get the desired result that $d\Delta(z)/dz > 0$.

This establishes the existence of a unique disclosure threshold in the disclosure sub-game between the certifier and the firm, which completes the proof of Lemma 1. 

Lemma 5. The function

$$
\gamma(t) = \tilde{\sigma} + \tilde{\lambda} t,
$$

is convex, symmetric and attains a minimum at zero; hence, it increases (decreases) when $t \geq 0$ ($t \leq 0$).

Proof. Note that $\gamma(t)$ is a symmetric function which can be rewritten as $\gamma(t) = \tilde{\lambda} t + \tilde{\lambda}$, Hence, $\gamma'(t) = \tilde{\lambda} t - \tilde{\lambda}$, The log-concavity of the normal distribution implies that its hazard rate is increasing (Bagnoli and Bergstrom, 2005) and that $\tilde{\lambda}$ is decreasing. Hence, $\gamma'(t) \geq 0$. Also, from Sampford (1953) we know that $\lambda'(t) > 0$. Hence, $\gamma'(t) = \lambda'(t) + \lambda'(t) > 0$. That the function $\gamma(t) = \tilde{\lambda} t + \tilde{\lambda} (t)$ attains its minimum at $t = 0$ follows from the result above that $\gamma'(t) \geq 0$.

We next proceed to the proof for Proposition 1.

Proof of Proposition 1. The proof of uniqueness is particularly involved, but the argument is as follows. We first observe that the profit function $\Pi(z)$ has a maximum. We then argue that the first order condition $\Pi'(z) = 0$ cannot be satisfied by more than one maximum. We prove this, by contradiction, by noting that if there were more than one local maximum, then either there would be a local minimum in between two local maxima or a continuum of maxima. To rule out both possibilities, we show that $\Pi(z)$ crosses zero only once. This means that maxima and minima cannot alternate, which yields a contradiction to any supposition of $\Pi(z)$ having multiple local maxima separated by at least one local minimum. The
profit function can be written as

\[ \Pi(z) = \Phi(-z)\Delta(z), \]

where

\[ \Delta(z) = \frac{\sigma_0^2}{\sigma_z} \int_{-\infty}^{z} \frac{\Phi(\frac{z}{\rho})\Phi(t)}{\Phi(z)} \, dt. \]

In the following comparative statics, we drop the constant \( \sigma_0^2/\sigma_z \) from \( \Delta(z) \). The first order condition yields

\[ \Pi_z(z) = \Delta(z)\Phi(-z) - \phi(z)\Delta(z) = 0, \]

which implies that

\[ \frac{\Delta(z)}{\Delta(z)} = \delta(z). \]

The second order condition reduces to

\[ \Pi_{zz}(z) = z\phi(z)\Delta(z) - 2\phi(z)\Delta(z) + \Phi(-z)\Delta(z). \]

Dividing by \( \Delta(z) \) and using (22) yields

\[ \frac{\Pi_{zz}(z)}{\Delta(z)} = z\phi(z) - 2\phi(z)\delta(z) + \Phi(-z)\Delta(z). \]

Some algebraic manipulation yields

\[ \frac{\Delta(z)}{\Delta(z)} = \frac{1}{\rho} \left( \frac{z}{\rho} \right) \gamma(z) - \lambda(z)\delta(z), \]

which follows from the fact that the first order condition implies that \( \Phi(z/\rho)/\gamma(z) = \Delta(z) \). Plugging (24) into (23) and diving by \( \Phi(-z)\gamma(z) \) yields

\[ \frac{\Pi_{zz}(z)}{\Delta(z)\Phi(-z)\gamma(z)} = \eta(z) = \lambda(z) + z - 2\delta(z) + \frac{1}{\rho} \left( \frac{z}{\rho} \right). \]

We next show that \( \eta(z) \) satisfies the single crossing property. There are two cases to consider \( \rho < 1 \) and \( \rho > 1 \). We only show the proof for \( \rho < 1 \), since the argument can be easily adapted to prove the result for \( \rho > 1 \). We proceed in two steps. We decompose \( \eta(z) \) as the sum of two functions

\[ \eta(z, \rho) = \frac{z - 2(\delta(z) - \lambda(z))}{m(z)} + \frac{1}{\rho} \left( \frac{z}{\rho} \right). \]

We then show that \( m(\cdot) \) is a decreasing function that crosses the \( x \)-axis at zero. Similarly, we show that \( n(z) \) single crosses the horizontal axis from above at a point \( z^* > 0 \), i.e., \( n(z^*) = 0 \), and that \( n'(z) \leq 0 \) for \( z \in [0, z^+] \).

These two facts, in turn, imply that there must exist a \( z^* \) such that \( \eta(z) \) is positive in \((-\infty, z^*)\) and decreasing in \([0, z^*]\). Also \( \eta(z) \) is negative in \((z^*, \infty)\). Taken together, these observations establish that \( \eta(z) \) is zero at a single point \( z^* \) located somewhere in \([0, z^+]\). Consider the function \( m(z) \). It is straightforward to verify that \( m(0) = 0 \). We next prove that \( m(z) \) is decreasing in \( z \). Notice that \( m(z) = -m(-z) \); hence, \( m'(z) \leq 0 \Rightarrow m'(-z) \leq 0 \). So, we can restrict attention to \( z \geq 0 \). Some simple algebra establishes that \( m(z) < 0 \).

Finally, we need to show that \( n(z) \) crosses zero from above at a single point \( z^+ > 0 \), and that \( n'(z) \leq 0 \) for \( z \in [0, z^+] \):

\[ n'(z) = \frac{1}{\rho} \left( \frac{z}{\rho} \right) - \lambda'(z). \]

Given that \( \lambda'(z) = -z\lambda(z) - \lambda^2(z) \), it follows that

\[ n'(z) = -\frac{\lambda(z)}{\rho} \left[ \frac{z}{\rho^2} + \frac{\lambda(z)}{\rho} \right] + \lambda(z)[z + \lambda(z)]. \]

But \( n(z^+) = 0 \Rightarrow (\lambda(z^+)/\rho) = \lambda(z^+) \), where \( z^+ \) must be positive (given that \( \rho < 1 \) and \( \lambda(z) < 0 \)). Hence,

\[ n'(z^+) = \lambda(z^+) \left[ z^+ - \frac{z^+}{\rho^2} \right] < 0, \text{ given } \rho < 1, \text{ and } z^+ > 0. \]

This proves the single crossing of \( n(z) \) for \( \rho < 1 \). For any \( z \leq z^+ \) we must have \( n(z) > 0 \), which implies that \( (\lambda(z)/\rho) = \lambda(z^+) \). Further, it is straightforward to establish that \( n'(z) < 0 \) for \( z \in [0, z^+] \). Thus, we have shown that \( m(\cdot) \) is decreasing, \( n(\cdot) \) single crosses the horizontal axis from above at \( z^+ \) and that \( n'(z) < 0 \) for \( z \in [0, z^+] \), as illustrated by Fig 3. These three results taken together establish that \( \eta(z) \) single crosses zero from above, which in turn implies single crossing of the second order conditions of the certifier’s maximization problem. There must, therefore, be a single value, \( z^* \), that maximizes the certifier’s
optimization program and this value satisfies the first order condition \( \Pi'(z^*) = 0 \). This completes the proof of uniqueness for \( \rho < 1 \). Adapting the above arguments, we can prove uniqueness for \( \rho \geq 1 \). □

**Proof for Corollaries 1 and 2.** These results follow from the usual comparative statics analysis and taking limits when required.

**Proof of Lemma 2.** Assume \( k_a > 0 \). Then, by standard results, we can write the price function as

\[
P(s, k_a) \equiv E\left(\bar{x}\mid s > k_a\right) = \beta s + \sigma \delta \left(\frac{k_a - \beta s}{\sigma}\right),
\]

where \( \beta = \sigma_x^2 / \sigma_t^2 \) and \( \sigma = \sqrt{\text{Var}(\bar{x} \mid s)} = \sigma_x \sqrt{1 - \beta} \). Next,

\[
\frac{\partial P(s, k_a)}{\partial s} = \beta \left(1 - \delta \left(\frac{k_a - \beta s}{\sigma}\right)\right) > 0,
\]

where the last inequality follows because \( \delta'(\cdot) < 1 \) (Heckman and Honore, 1990). Also,

\[
\frac{\partial^2 P(s, k_a)}{\partial s \partial k_a} = \beta^2 \delta \left(\frac{k_a - \beta s}{\sigma}\right) > 0,
\]

where the last equality follows from \( \delta'(\cdot) > 0 \) (see e.g., Sampford, 1953). To obtain the lower bound of \( P(s, k_a) \), observe that

\[
\lim_{s \to -\infty} \sigma \delta \left(\frac{k_a - \beta s}{\sigma}\right) = \lim_{s \to -\infty} \frac{k_a - \beta s}{\sigma} = \lim_{s \to -\infty} (k_a - \beta s).
\]

Hence, \( \lim_{s \to -\infty} P(s, k_a) = k_a \). □

**Proof for Lemma 3.** Follows from standard results. □

**Proof of Proposition 2.** To simplify the notation in subsequent proofs, we normalize \( \sigma_x = 1.^{26} \)

(i) **Existence of a unique threshold for a given fee** \( c_a > 0 \): Fix any given fee \( c_a > 0 \). To prove the existence of the unique threshold \( k_a \) in the subgame between the firm and the certifier for this fixed fee \( c_a \), monotonicity properties of \( \delta(t) \) and \( \lambda(t) \) allow us to establish that

\[
\Delta_a(t) = (1 - \sigma^2) \frac{t + \sigma \delta(t \sigma) + \lambda(t) - \phi(0)}{\Phi(t)}
\]

increases in \( t \). Together with the fact that \( \Delta_a(t) \) goes from zero to infinity, this monotonicity of \( \Delta_a(t) \) establishes the desired result.

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26 This normalization of \( \sigma_x = 1 \) is without loss of generality.
(ii) Disclosure probability decreasing in disclosure precision: Let $k_a$ be the disclosure threshold for a given certification fee in this ex ante disclosure regime. Applying the Implicit Function Theorem to Eq. (13) yields

$$\frac{\partial k_a}{\partial \sigma} = -\frac{\partial \Delta_a(k_a)}{\partial \sigma},$$

We establish the desired result by showing that $\partial k_a/\partial \sigma < 0$. Since $\sigma$ increases in $\sigma_c$ and $\Delta_a'(k_a) > 0$, to establish that $\partial k_a/\partial \sigma_c < 0$ we only need to prove that $(\partial/\partial \sigma)\Delta_a(k_a) > 0$. By standard results, one can write $\Delta_a(\cdot)$ as

$$\Delta_a(k_a) = (1 - \sigma^2)k_a + \sigma \delta(k_a) + \lambda(k_a) - \Phi(0).$$

Differentiating the above equation with respect to $\sigma$ yields $(\partial \Delta_a(k_a))/\partial \sigma = -2r + \delta(r) + r\delta'(r)$, where $r \equiv \sigma k_a$. It is well known that $\lim_{r \to \infty} \delta(r) = 1$ and that $\lim_{r \to -\infty} (\delta(r) - r) = 0$ (see Barrow and Cohen, 1954). Hence, $\lim_{k_a \to -\infty} (\partial/\partial \sigma)\Delta_a(k_a) = 0$. We next show that

$$\frac{\partial \Delta_a(k_a)}{\partial \sigma} = -2r + \delta(r) + r\delta'(r) > 0$$

for all finite $k_a$. We proceed by contradiction. Suppose there exists a finite $k_a$ such that

$$\frac{\partial \Delta_a(k_a)}{\partial \sigma} = -2r + \delta(r) + r\delta'(r) \leq 0$$

$$\Rightarrow r\delta'(r) \leq 2r - \delta(r).$$

Differentiating $(\partial^2/\partial \sigma^2)\Delta_a(k_a) = -2 + 2\delta(r) + r\delta'(r)$, but since $\delta'(r) = \delta'(r)(\delta(r) - r) + \delta(r)(\delta(r) - 1)$, it follows that

$$\frac{\partial^2 \Delta_a(k_a)}{\partial \sigma^2} = -2 + 2\delta(r)(\delta(r) - r) + r\delta'(r)(2\delta(r) - r) - \delta(r)r.$$

From Sampford (1953), we know that $2\delta(r) - r > 0$. Substituting this inequality into the above equation yields

$$\frac{\partial^2 \Delta_a(k_a)}{\partial \sigma^2} \leq -2(1 - \delta(r)r + r^2),$$

but Barrow and Cohen (1954) proved that $1 - \delta(r)r + r^2 > 0$. Hence $(\partial^2/\partial \sigma^2)\Delta_a(k_a) < 0$ for all finite $k_a$, which implies that $(\partial/\partial \sigma)\Delta_a(t) < 0$ for all $t > k_a$. But this contradicts

$$\lim_{t \to \infty} \frac{\partial \Delta_a(t)}{\partial \sigma} = 0.$$  

(26)

We thus conclude that $(\partial/\partial \sigma)\Delta_a(k_a) > 0$ for all finite $k_a$.

**Proof of Proposition 3.** To establish part (ii), we must show that $k_a^* > 0$. We show this by contradiction. Fix any positive finite $\sigma_c$. Assume that $k_a^* \leq 0$. To contradict this we will show that the function $\Pi(k_a)$ is log-concave, hence single peaked, and that $\Pi(0) > 0$, which contradicts $k_a^* \leq 0$. Fix a threshold $k_a^* < 0$. We can write

$$\Pi(k_a^*) = \Phi(-k_a^*) \left[ \int_0^\infty \frac{t}{\Phi(\frac{k_a^*+1-\beta}{\sigma})} \Phi\left(\frac{t-\beta k_a^*}{\sigma}\right) dt - E[\hat{x} k_a^*] \right].$$

Some algebra yields

$$\Pi(k_a^*) = \Phi(-ah) \frac{\partial}{\partial h} \Pi(a) \left[ a\Phi(a) + \Phi(a) \right],$$

where $a = \beta k_a^*/(1-\beta)$ and $h = (\sqrt{1-\beta})/\beta, l = (1-\beta)/\beta$. Now, $\Pi(k_a^*)$ is the product of two log-concave functions, and hence, is itself log-concave:

$$\Pi'(k_a^*) = \frac{\beta \Pi(k_a^*)}{\sqrt{1-\beta}} \left[ -h \Phi(ah) \Phi(-ah) + l \Phi(ah) \Phi(-ah) \right] + \frac{\Phi(a)}{a \Phi(a) + \Phi(a)}.$$

Evaluating the above equation at $k_a = 0$, it can be shown that $\Pi'(0) < 0$, which yields the result that $k_a^* > 0$. Part (iv) is established by noting that the certifier’s expected profits

$$\Pi(k_a^*) = \text{Pr}(\hat{x} > k_a^*) \Delta_a(k_a^*)$$

and establishing that each one of these two components is increasing in $\sigma_c$. That the second component $\Delta_a(k_a^*)$ is increasing in $\sigma_c$ was established in the last proof because of our result that $(\partial/\partial \sigma)\Delta_a(k_a^*) > 0$, i.e., $k_a^*$ increases in $\sigma$, hence it must
increase in \( \sigma_x \) too. Next, applying implicit function theorem and using the result that \( \frac{\partial k^*_n}{\partial \sigma_x} < 0 \) yield the result that \( \Pr(\tilde{x} > k^*_n) \) also increases in \( \sigma_x \).

**Proof of Proposition 4.** In the ex ante regime, we know from Proposition 3 that \( \lim_{\tau_e \to 0} \Pr(\tilde{x} \geq k^*_n) = \frac{1}{2} \) and that \( \lim_{\tau_e \to \infty} \Pr(\tilde{x} \geq k^*_n) = 0.241 \). Also, from Corollary 1 we know that in the ex post regime the probabilities of disclosure are given by

\[
\lim_{\tau_e \to 0} \Pr(\tilde{s} \geq k^*_n) \approx 0.765 \quad \text{and} \quad \lim_{\tau_e \to \infty} \Pr(\tilde{s} \geq k^*_n) \approx 0.241.
\]

By continuity, this means that for sufficiently small \( \tau_e \), the probability of disclosure is higher under the ex post regime than under the ex ante regime. Of course, as \( \tau_e \to \infty \) the probability of disclosure under ex ante and ex post regimes is the same. □

**References**


