The Public Market Equivalent and Private Equity Performance

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Abstract

We show that the public market equivalent (PME) approach [Kaplan and Schoar (2005)] is equivalent to assessing the performance of PE investments using Rubinstein’s dynamic version of the capital asset pricing model [Rubinstein (1976)]. Two insights follow. First, we do not have to compute betas of PE investments, and any changes in PE cash flow betas due to changes in financial leverage, operating leverage, or the nature of the businesses are automatically taken into account. Second, the public market index used in evaluations should be the one that best approximates the wealth portfolio of the investor who is considering the PE investment opportunity.

Keywords: Private equity, public market equivalent, PME, log utility CAPM, ex-post performance evaluation, generalized method of moments.

JEL Classification: G12, G23, G24, G31.
We consider the practical problem of evaluating the performance of private equity ("PE") funds, such as buyout ("BO") or venture capital ("VC") funds. These funds hold privately-held companies, without quoted market prices. Hence, the funds do not have regular monthly or quarterly quoted market returns, which are usually needed to evaluate financial performance, and their performance must be evaluated from their cash flows alone. A natural starting point is to calculate the funds’ internal rate of return ("IRR"). The IRR, however, has several problems: It is an absolute performance measure that does not adjust for the market return or the risk of the investments. The IRR implicitly assumes investors can reinvest cash flows at the IRR rate. It may not exist, and it may not be unique. Moreover, investors in PE funds ("LPs") have been concerned that PE funds manipulate their IRRs by deliberately choosing the timing and size of their investments. Essentially, these problems arise because the IRR is not grounded in a valuation theory, and in their evaluation of the Kauffman Foundation’s VC investment program, Mulcahy, Weeks, and Bradley (2012) recommend that LPs “Reject performance marketing narratives that anchor on internal rate of return (IRR), top quartile, vintage year, or gross returns,” and instead “Adopt PME as a consistent standard for VC performance reporting.”

The PME (short for “public market equivalent”) also measures performance from cash flows alone, like the IRR.\(^1\) It is calculated as follows: Let \(X(t)\) be the cash flow from the PE fund.
fund to the LP at time \( t \). This cash-flow stream is separated into its positive and negative parts, called distributions, \( \text{dist}(t) \), and capital calls, \( \text{call}(t) \). A distribution is a cash flow that is returned to the LP from the PE fund (net of fees) after the fund successfully sells or recapitalizes a company. Capital calls are the investments by the LP into the fund (including the payment of management fees). Each of these distributions and capital calls is then discounted using realized market returns, and the ratio of the two resulting valuations is the PME:

\[
PME = \frac{\sum_t \text{dist}(t)}{1+r_M(t)} \bigg/ \frac{\sum_t \text{call}(t)}{1+r_M(t)} .
\]

In this equation, the sum runs over the life of the fund, and \( r_M(t) \) is the realized (not expected) market return from the inception of the fund, at \( t = 0 \), to the time of the distribution or capital call, at time \( t \). A PME greater than one means that the value of the distributions exceeds the cost of the capital calls, and the LP has profited from the investment in the fund.

From a standard valuation perspective, though, the PME calculation looks strange. In the standard capital asset pricing model ("standard CAPM") the present value ("PV") is calculated by discounting cash flows using the CAPM discount rate:

\[
r_{CAPM} = r_F + \beta (E[r_M] - r_F) ,
\]

where \( r_F \) is the risk-free rate, beta measures the cash flow’s systematic risk,\(^2\) and \( E[r_M] - r_F \)

\(^2\)Formally, given as \( \beta = \text{cov}(r_M, r_{PE})/\text{var}(r_M) \).
is the expected market risk premium. The PV is then:

$$ PV = \sum_t \frac{E[X(t)]}{(1 + r_{CAPM})^t}. $$

(3)

This standard CAPM is the well-known textbook approach to valuation. Investments that earn higher returns than their cost of capital, $r_{CAPM}$, have positive PVs.

The PME in equation (1) appears fundamentally different from the standard CAPM in equation (3). The standard CAPM has no role for realized market returns, which are used as discount rates in the PME calculation. The discount rate in the standard CAPM is based on the expected market risk premium and is the same for all future cash flows. In contrast, there is no beta in the PME calculation, and different cash flows are discounted at different rates as the market return varies.

Informally, this PME calculation has been justified as the relative performance of the PE investment compared to a strategy where the LP reinvests distributions into the market. Alternatively, when beta equals one, the discount rate in the standard CAPM is the expected market return, which looks similar to the discount rate in the PME calculation (ignoring the difference between expected and realized market returns). This intuition led Kaplan and Schoar (2005) to argue that the PME is valid when the beta equals one. But the beta may well differ from one, particularly for buyout funds, which invest in highly levered transactions. In this case, how can one interpret the PME?

To better understand the properties of the PME, we derive a rigorous economic foundation. We show that the PME is equivalent to evaluating PE investments under the Rubinstein (1976) dynamic version of the capital asset pricing model (CAPM). This foundation is im-
portant for several reasons: It means that the PME does not have the kinds of problems that the IRR suffers from, and it gives more credibility to the PME as a performance measure. This foundation allows us to derive the formal properties of the PME, and it turns out that these properties are surprisingly useful, but also counterintuitive, relative to the standard CAPM. We find that the PME is robust. The PME does not require a beta of one, and it is valid regardless of the investment’s beta, even if this beta changes over the life of the investment. In fact, with the PME, investors can evaluate risk-adjusted performance without explicitly calculating any betas or even knowing the risk of the underlying investments. We find that the market index used to discount the cash flows in the PME should ideally approximate the return to the investor’s overall wealth portfolio, not the specific risk of the PE investments. This is different from the standard CAPM, where an investment is evaluated relative to other comparable investments with similar risks. In contrast, the PME values the investment as an integrated part of the investor’s overall portfolio.

Formally, we show that the PME is a valid performance measure under three conditions, neither of which is particularly controversial nor restrictive: (1) markets are frictionless and the “law-of-one-price” holds; (2) the LP has logarithmic preferences;\(^3\) and (3) the return on the LP’s total wealth portfolio equals the return on the public market.

An important limitation of the PME is that it is better suited for evaluating past performance, ex-post. The risk adjustment of the PME arises from a complicated covariance between the market return and the investment cash flows, as we show below, but this co-

\(^3\)Or more generally, has Kreps-Porteus (1978) preferences with relative risk aversion of one
variance may be difficult to forecast ex-ante. In contrast, the standard CAPM uses the correlation between the market and the investment’s return, which is easier to forecast. Hence, in this note, we focus only on evaluating past performance of realized cash flows, not evaluating potential future investments based on their projected cash flows.

Our discussion is organized as follows. Section 1 describes the stochastic discount factor approach for valuing streams of future cash flows and the Rubinstein’s (1976) CAPM. Section 2 has a numerical example that illustrates the similarities and differences between the two valuation methods. In Section 3, we discuss the PME and its relation to the Rubinstein’s (1976) CAPM. Section 4 concludes. A reader who is not interested in the formal details can proceed directly to Section 4.

1 Present Value of Risky Cash Flows: Stochastic Discount Factor Approach

Starting at time $t = 0$, an investor can value a risky future cash-flow stream as follows. At each point in time, $t$, a state of the world is realized, denoted $s_t$, with probability $p(s_t)$, as assessed by the investor. The stochastic discount factor (“SDF”), denoted $m(s_t)$, then gives the investor’s valuation (at time 0) of a future cash flow of one dollar at time $t$, in state $s_t$, scaled by its probability. From this basic building block, the investor can value any risky cash-flow stream. For example, the value of the cash flow $X(s_t)$ is:

$$ PV = \sum_t \left[ \sum_{s_t} p(s_t)m(s_t)X(s_t) \right] = \sum_t E \left[ m(s_t)X(s_t) \right]. \quad (4) $$
Here, $m(s_t)$ is the stochastic discount factor that discounts the cash flow at time $t$ in state $s_t$, and the investor’s PV of the cash flow in this state is $p(s_t)m(s_t)X(s_t)$.

The SDF is the foundation of a valuation model.\footnote{The Stochastic Discount Factor framework for valuing risky cash flows was developed by Hansen and Richard (1987). Two excellent discussions of this framework are in the textbooks by Cochrane (2001) and Skiadas (2009).} Any set of prices or valuations that are internally consistent (formally, when the “law-of-one-price” holds) can be generated by some SDF. For a risk-averse investor, cash flows that are larger when the market is down are more valuable, because they insure the investor against downturns. Since the SDF values cash flows across states, the SDF should also be larger in those states where the market return is poor.

To illustrate, the SDF representation of the standard CAPM is:

$$m(s_t) = a - b [1 + r_M(s_t)]$$

(5)

where $r_M(s_t)$ is the return on the market from time 0 to time $t$, in state $s_t$; and $a$ and $b$ are constants that reflect the risk-free rate, the market risk, and the market price of risk (see Dybvig and Ingersoll (1982) and Hansen and Richard (1987)). An important advantage of the standard CAPM is that this linear SDF implies that the risk of any investment is fully summarized by its correlation with the market, as measured by $\beta$, and it is unnecessary to track individual cash flows across different states. This advantage is a main reason why the standard CAPM is used for most valuations done in practice.
1.1 Rubinstein CAPM

Rubinstein (1976) derived an alternative capital asset pricing model (“Rubinstein CAPM”). Suppose that markets are frictionless, and that an investor is infinitely lived and chooses lifetime consumption and investment plans, subject to budget constraints, to maximize lifetime expected utility as represented by the time-separable logarithmic (log) utility function:\(^5\)

\[
U (c(s_t)) = \sum_t \beta^t E \left[ \ln (c(s_t)) \right].
\]

(6)

With log-utility preferences, the investor’s SDF is \(m(s_t) = [c(s_t)/c(0)]^{-1}\), where \(c(s_t)/c(0)\) is the investor’s consumption growth from time 0 to state \(s_t\) at time \(t\). With log utility the investor consumes a constant fraction of total wealth, so \(c(s_t)/c(0) = 1 + r_W(s_t)\), where \(r_W(s_t)\) is the return on the investor’s total wealth. Assuming \(r_W = r_M\), it follows that:

\[
m(s_t) = \frac{1}{1 + r_M(s_t)}.
\]

(7)

This non-linear SDF is different from the linear SDF from the standard CAPM, so it is a different valuation model. Equation (4) implies that cash flows are valued by discounting with the realized market returns, and the PV of \(X(s_t)\) is:

\[
PV = \sum_t E \left[ \frac{X(s_t)}{1 + r_M(s_t)} \right].
\]

(8)

\(^5\)These formal assumptions may appear restrictive, but they should be interpreted as general approximations of the economic environment, and they seem no more contrived than the assumptions required to derive the standard CAPM. For example, the standard CAPM assumes, among other things, that the investor is myopic with a one-period horizon and that risk is measured by variance alone. Moreover, as Dybvig and Ingersoll (1982) point out, the standard CAPM implies that the SDF takes negative values when the market return is sufficiently high, meaning that a contingent claim that pays a positive amount in these states has a negative value.
Under both the Rubinstein and standard CAPM, idiosyncratic risks are diversified away and only systematic risk is priced, although the two models differ slightly in the way they price this systematic risk. Under the standard CAPM, the systematic risk is summarized by beta. A higher beta gives a larger discount rate, and this constant discount rate is used for all cash flows. In contrast, in the Rubinstein CAPM, even when the systematic risk remains constant, the discount rate varies with the market return. Cash flows that pay off when the market is low are discounted at a lower rate, giving them a higher PV, as they should have.

To formally see the difference in the pricing of systematic risk, under the Rubinstein model, the PV of a one-period cash flow is $E\left[\frac{X}{1+r_M}\right]$, which also equals $E[X] \times E[1/(1+r_M)] + COV[X,1/(1+r_M)]$. The last term is the covariance between the cash flow and the (inverse) market return. Hence, discounting with realized market returns implicitly captures systematic risk, albeit measured slightly differently than in the standard CAPM. This is the complicated covariance term, mentioned earlier, which is more difficult to project, ex-ante.

1.2 Comments

A few comments are in order. We assume $r_W = r_M$, but we want to emphasize the distinction between these two returns. The valuation is derived using $r_W$, which is the return on the investor’s wealth portfolio. We assume that this return equals the return on the market portfolio, where the market portfolio is defined broadly to contain all stocks, bonds, and other assets. Formally, this equality holds, in equilibrium, when markets are complete and all investors have the same beliefs and preferences, with a constant relative risk aversion.
of one. More generally, without complete markets and with heterogeneous agents, the two
returns may differ and the equality \( r_M = r_W \) may fail. In this case, an investor with log
utility should discount cash flows using \( r_W \), instead of \( r_M \), as the stochastic discount rate.

While conceptually straightforward, implementing this approach in practice requires taking
a stand on what is a reasonable proxy for the wealth portfolio of the investor evaluating
the performance of the PE investment. In general, the wealth portfolio contains publicly-
traded stocks, bonds, and other securities. When the wealth portfolio has significant amounts
invested in illiquid assets that are not publicly traded, it is necessary to estimate the returns
on those assets, for example based on the valuation of experts. Further, ex-post performance
measurement necessarily has to take into account the extent to which the events that hap-
pened were due to luck or the lack of it, and we refer to Kaplan and Schoar (2005) for a
discussion of the necessary statistical framework.

Several other SDFs have been proposed for evaluating PE and VC performance. Driessen,
Lin, and Phalippou (2012) propose a GMM estimator that implicitly uses
\[ m = \frac{1}{\prod (1 + r_F + \alpha + \beta r_M)} \]
where \( r_F \) is the risk free rate and \( r_M \) is the return on the market, as a measure of \( r_W \)
for some constants \( \alpha \) and \( \beta \), and Ang, Chen, Goetzmann, and Phalippou (2013) calculate
the parameters of this SDF that implies that investors break even. Korteweg and Nagel
(2013) value VC investments using a SDF that is a nonlinear function of selected stock index
returns, as in Bansal, Hsieh and Viswanathan (1993) and Glosten and Jagannathan (1993).
Different SDFs imply different risk adjustments, and trying to determine which of these
SDFs that provides the best risk adjustment is an important empirical question that goes
beyond the scope of this note.

2 Numerical Examples

The properties of the PME can be counterintuitive, and it is illustrative to work through a few concrete examples to understand how it works. First, we value a cash flow using both the PME and the standard CAPM to show their similarities and differences. Then, we leverage the cash flow and value it again. Perhaps surprisingly, leverage does not artificially inflate PMEs.

Let the market return over a three-year period be given by the binomial tree in Panel A of Table 1. In each year, with a 50% probability, the market appreciates by 20% or depreciates by 10%. This is obviously a simplified market structure, but it helps to highlight the workings of the PME, and nothing hinges on this simplification. Under the Rubinstein CAPM, the market determines the risk-free rate and the market risk premium. The risk-free rate is $r_F = 2.86\%$ and the market risk premium is $E[r_M] - r_F = 2.14\%$.

Consider now a PE investment that generates the cash flows given in Panel B of Table 1. Initially, at time 0, an LP invests $1,000 into the PE fund. During the first year, the fund matures, and after the second year, depending on the market return, it returns either $405$, $585$, or $845$. After the third year, the fund returns either $364.50$, $526.50$, $760.50$, or $1,098.50$. These cash flows are net of fees, and there is no need to model fees explicitly.

The risk-free rate is calculated using $(1 + r_F)^{-1} = E[m]$, which holds generally for SDFs. Given the risk-free rate, the market risk premium can be calculated directly as $E[r_M] - r_F$.

These cash flows are constructed by assuming that the PE fund’s underlying valuation increases by 30% when the market appreciates, that it decreases by 10% when the market depreciates, that half of the PE fund’s value is returned after the second period, and that the remainder is returned at the end. But this underlying structure is unimportant for the valuation, and the method can be applied to any arbitrary cash
The probabilities of each branch in the tree are given in Panel C of Table 1, and the average cash flows, calculated using these probabilities, are in Table 2.

Valuing this cash flow using the Rubinstein CAPM is straightforward. The cash flows are discounted using the observed market returns and are then averaged. The valuations are in Table 2, and the total PV is $107.68.

Using the standard CAPM is more complicated. First, we need the average cash flows in each year, $E[X]$, as reported in Table 2. Then, we need the beta. Below, we show that the beta is 1.29, but in practice this beta must be estimated from publicly-traded comparable companies. The standard CAPM discount rate is then: 

$$r_{CAPM} = r_F + \beta \times (E[r_M] - r_F) = 2.86\% + 1.29 \times 2.14\% = 5.64\%.$$  

Finally, we discount the expected cash flows using this discount rate, and the PV is $106.99. This PV is close to the PV that we found using the Rubinstein model. The two risk adjustments are similar, and the two models produce almost identical valuations in this setting.

### 2.1 Valuation using SDFs

We can also value the cash flow using just the SDFs, shown in Table 3. The Rubinstein SDF is simple to calculate, and it is $m = 1 / (1 + r_M)$, as given in Panel B of Table 3. Again, the standard CAPM is more complicated. The SDF is $m = a - b(1 + r_M)$, where $a$ and $b$ are determined by the risk-free rate and the market risk premium. For each year we can solve for $a$ and $b$.\footnote{The $a$ and $b$ are calculated as follows. In each year, the following equations hold for all SDFs: $E[m] = (1 + r_F)^{-1}$ and $E[m(1 + r_M)] = 1$. Substituting the linear expression for $m$ into these two equations results in two equations with two unknowns, $a$ and $b$, which are solved by inverting a two-by-two matrix.} In year one, we have $a = 1.944$ and $b = 0.926$; in the second year, $a = 1.871$
and $b = 0.840$; in the third year, $a = 1.801$ and $b = 0.762$. Using these values, we calculate the SDF as given in Panel A of Table 3. Using this SDF, we calculate the PV by multiplying each cash flow with the corresponding SDF and taking the average, using the probabilities in Panel C of Table 1. As they should be, the resulting PVs are numerically identical to those in Table 2, with PVs of $107.68$ and $106.99$. In fact, for the standard CAPM, we determined the beta of 1.29 from this SDF valuation by solving for the beta that implies the same PV.

### 2.2 Effects of Leverage

Since the Rubinstein CAPM is valid regardless of the investment risk, it is also robust to changes in leverage. This means that PE funds that don’t create actual value cannot inflate their PMEs to exceed one by simply leveraging up.\(^{10}\) This may be surprising. To see that leverage does not matter, consider an investor that invests $1,000 in the market, but only puts up half of this amount and borrows the remaining $500. For concreteness, let this be a three-year investment, and let the debt be three-year bullet debt, so the principal and compound interest are repaid at year three. With a three-year risk-free rate of 8.82%\(^{11}\), the investor’s final debt payment is $544.09, and the resulting cash flows are given in Panel A of Table 4. As above, we can value this leveraged investment in the market using the

\(^{10}\)A formal derivation of this result is as follows: Let $PV_X$ be the PV of the PE cash flows, $X$. The debt is fairly priced, so $PV_D = 0$. The PV of the levered deal is $PV_L = PV_X + PV_D = PV_X$, as illustrated in this example. To see the effect of leverage on the PME, divide the debt’s cash flow into its positive and negative parts, denoted $DP$ and $DN$. Fair pricing of the debt implies that $PV_{DP} - PV_{DN} = 0$ and $PV_{DP} = PV_{DN}$. When the unlevered $PME = PV_{DIST}/PV_{CALL}$, then the leveraged $PME_L = (PV_{DIST} + PV_{DP})/(PV_{CALL} + PV_{DN})$, so increasing leverage brings the PME closer to one.

\(^{11}\)The three-year risk-free interest rate is calculated by valuing the cash flow that pays $1 in each branch at year three. Using the Rubinstein SDF, the value is $0.9190$, and the risk-free rate is found by solving $(1 + r_F)^{-1} = 0.9190$, implying $r_F = 8.82\%$. 

\[12\]
Rubinstein CAPM. Each cash flow is discounted by the corresponding market return and averaged, using the probabilities in Panel C of Table 1. As it should be, the total PV of this leveraged investment is exactly $0, as shown in Panel C of Table 4.

Alternatively, consider a leveraged investment in the PE cash flow. Again, let the investment be $1,000 where half is financed with risk-free bullet debt, as above. The resulting cash flows are given in Panel B of Table 4. In year zero, the investor’s initial investment is reduced to $500. In years 1 and 2 the cash flows are identical to the cash flows from the original PE investment, from Panel B of Table 1. In year 3, these original cash flows are reduced by $544.09 to service the debt. Panel C of Table 4 shows the PV of the resulting levered PE cash flow of $107.68, which is identical to the PV of the unlevered cash flow.

Overall, these examples show that the Rubinstein CAPM is robust to changes in risk and leverage. PE funds cannot artificially inflate their performance by leveraging a simple investment in the market or by simply leveraging their existing deals, assuming correctly priced debt.

3 Public Market Equivalent

The PME is closely related to the Rubinstein CAPM. With a sufficient number of observed deals for each PE fund, the expectation in equation (8) can be estimated using a generalized method of moments (“GMM”) estimator that replaces the expected values with the realized cash flows. Specifically, index the fund’s deals by $i = 1, 2, ..., N$, and separate each deal’s

\footnote{The cash flow at year three may now be negative, but we assume that the investor can commit to honor the resulting liability, so the debt remains risk free. The valuation remains unchanged, however, with risky debt as long as this risk is fully anticipated and priced initially.}
cash flows into capital calls, denoted \( call_i(t_i) \), and distributions, \( dist_i(t_i) \). Let \( DIST \) and \( CALL \) denote the numerator and denominator in the PME definition, so \( DIST \) is:

\[
DIST = \sum_i \frac{dist(t_i)}{1 + r_M(t_i)} = \sum_i \frac{dist_i(t_i)}{1 + r_M(t_i)},
\]

(9)

where \( dist(t) \) equals the sum of the distributions at time \( t \) across deals. For a sufficiently large \( N \) (and under suitable regularity conditions ensuring the law of large numbers), the average of the observed distributions converges to their expected value, which equals the PV of the distributions under the Rubinstein CAPM:

\[
\frac{1}{N} \cdot DIST \sim E \left[ \frac{dist(s_t)}{1 + r_M(s_t)} \right] = PV_{DIST}.
\]

(10)

A similar argument holds for the capital calls, and after canceling the \( 1/N \) terms, the PME equals:

\[
PME = \frac{DIST}{CALL} \sim \frac{PV_{DIST}}{PV_{CALL}}.
\]

(11)

The asymptotic sampling distribution of the PME can be computed with suitable further assumptions.

Hence, under the conditions outlined above, the Kaplan and Schoar (2005) PME measure is a consistent estimate of the PV of the distributions divided by the PV of the capital calls. A PME greater than one implies that the investment has been profitable for the LP.

\[\text{To simplify the notation, we assume that each deal has only a single capital call and distribution, but the analysis holds with an arbitrary number of cash flows, although the notation becomes more cumbersome. Specifically, let } i(c), \text{ for } c = 1, ..., C(i) \text{ index the individual cash flows associated with deal } i. \text{ Replacing } \sum_i \frac{dist_i(t_i)}{1 + r_M(t_i)} \text{ with } \sum_i \left( \sum_c \frac{dist_{i(c)}(t_{i(c)})}{1 + r_M(t_{i(c)})} \right) \text{ and noting that } E \left[ \left( \sum_c \frac{dist_{i(c)}(t_{i(c)})}{1 + r_M(t_{i(c)})} \right) \right] = PV_{dist} \text{ generalizes the derivation to an arbitrary number of cash flows for each deal.} \]
4 Summary and Conclusion

The paper provides a rigorous theoretical justification for the public market equivalent (PME) measure of private equity performance. A main advantage of the PME is that it is simple to calculate and apply. This simplicity arises because the PME is calculated without explicitly relying on the risk of the investment (such as its beta), and the PME is valid regardless of this risk. It does not require different treatments of cash flows associated with capital calls and distributions, despite their different properties. The PME does not rely on the LP following any particular trading strategy, such as reinvesting distributions dollar for dollar into the market portfolio. As a valid economic measure, it is robust to manipulation.

Our theoretical justification follows from the observation that the PME can be viewed as a PV calculation under the Rubinstein CAPM (1976). It follows that the more appropriate public-market return to use when computing the PME is the return on the LP’s overall wealth portfolio.

Our analysis also clarifies the difference between two fundamentally different ways to assess risk-adjusted performance. In the standard CAPM, expected cash flows are discounted with expected market returns, and the risk adjustment is done by increasing this discount rate as a function of the beta, which reflects the covariance between the returns on the investment and the market. Under the Rubinstein (1976) CAPM, risk-adjusted performance is calculated by discounting with realized market returns, which implicitly captures systematic risk, because $E[X/(1 + r_M)] = E[X] \times E[1/(1 + r_M)] + COV[X, 1/(1 + r_M)]$.

The PME has some important limitations. It is more useful as an ex-post measure
of past performance than for evaluating future investment opportunities, ex-ante, because correlations between cash flows and future market (inverse) returns are more difficult to project. Further, the PME provides the value at the margin, which is relevant for making a small additional investment in PE. This value at the margin, however, does not provide sufficient information when evaluating substantial asset-allocation decisions to PE, which also requires a consideration of the illiquidity of the PE investment and the investment capacity. Further, with larger PE allocations, the wealth portfolio of the investor evaluating the PE investments also changes, which affects the benchmark public-market return that is used in the calculation of the PME measure itself. One way to address this issue is to use the return on a portfolio of publicly-traded securities that mimics the return on the investor’s wealth portfolio, and add a suitable premium as compensation for the illiquidity of the PE investment, as in Driessen, Lin and Phalippou (2012). The PME may be less flexible for adjusting for other specific risk exposures than, for example, the multi-factor CAPM. If PE returns reflect a general premium earned by small-value stocks (as suggested by Phalippou 2013) it may also be useful to compare the PME of the PE investment with the PME of such publicly-traded alternatives.

As a final comment, several different measures have been proposed for evaluating PE performance. At this point, it is too early to definitively recommend any single model or measure. A more reasonable recommendation is to evaluate performance in several ways, and when the conclusions differ sharply, probe deeper to understand the reasons.
References


Mulcahy, Diane, Bill Weeks, and Harold Bradley (2012) “We Have Met the Enemy ... And He is Us,” *working paper: Kauffman Foundation*


**Table 1: Market returns and PE cash flow.** Panels A and B show binomial trees with the market return and the PE cash flows. Each tree start at time zero and span three years, with each branch having a 50% probabilities. The resulting probability of each branch, used when calculating averages, are in Panel C.

**Panel A: Market Return: \(1+r_{M}(s_t)\)**

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<th>2</th>
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**Panel B: PE Cash Flow: \(X(s_t)\)**

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**Panel C: Probability: \(p(s_t)\)**

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<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/8</td>
<td>1/4</td>
<td>1/2</td>
<td>3/8</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1/2</td>
<td>1/2</td>
<td>3/8</td>
</tr>
<tr>
<td></td>
<td>1/4</td>
<td></td>
<td></td>
<td>1/8</td>
</tr>
</tbody>
</table>
TABLE 2: Present Value. The table shows average cash flows of the PE investment, and its present values, calculated using both the standard and Rubinstein CAPMs.

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>E[X(s_i)]</td>
<td>-$1,000.00</td>
<td>0.00</td>
<td>605.00</td>
<td>665.50</td>
<td>270.50</td>
</tr>
<tr>
<td>Standard CAPM</td>
<td>-$1,000.00</td>
<td>0.00</td>
<td>542.37</td>
<td>564.62</td>
<td>106.99</td>
</tr>
<tr>
<td>Rubinstein CAPM</td>
<td>-$1,000.00</td>
<td>0.00</td>
<td>542.53</td>
<td>565.14</td>
<td>107.68</td>
</tr>
</tbody>
</table>
TABLE 3: Stochastic Discount Factors (SDFs). The two panels show binomial trees, corresponding to the binomial trees for the market return and cash flows in Table 1. Panel A shows the SDF under the standard CAPM, calculated as $m = a - b R_M$, where $a$ and $b$ are constant given in the text. Panel B shows the SDF under the Rubinstein CAPM, calculated as $m = 1/(1 + R_M)$.

Panel A: SDF for Standard CAPM: $m = a - b (1 + r_M(s_t))$

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.484$</td>
<td>$0.662$</td>
<td>$0.833$</td>
<td>$0.814$</td>
</tr>
<tr>
<td></td>
<td>$1.000$</td>
<td>$0.964$</td>
<td>$1.060$</td>
<td>$1.060$</td>
</tr>
<tr>
<td></td>
<td>$1.111$</td>
<td>$1.191$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1.191$</td>
<td>$1.246$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: SDF for Rubinstein CAPM: $m = 1 / (1 + r_M(s_t))$

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.579$</td>
<td>$0.694$</td>
<td>$0.833$</td>
<td>$0.772$</td>
</tr>
<tr>
<td></td>
<td>$1.000$</td>
<td>$0.926$</td>
<td>$1.111$</td>
<td>$1.029$</td>
</tr>
<tr>
<td></td>
<td>$1.111$</td>
<td>$1.235$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1.235$</td>
<td>$1.372$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 4: Levered Cash Flows. Panels A and B show cash flows from levered market and PE investments. Each levered cash flow is given by a three-year investment of $1,000 that is financed with $500 debt. For simplicity, the debt is bullet debt, which is repaid in full, with compound interest, in year 3, when the investment is also liquidated. The three-year risk-free interest rate is 8.82%, and the total debt payment in year 3 is $544.09. Panel C shows valuations of the two cash flows, using the Rubinstein CAPM.

Panel A: Cash Flow for Levered Market Investment

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$500</td>
<td>$0</td>
<td>$0</td>
<td>$500</td>
</tr>
<tr>
<td>$1,184</td>
<td>$0</td>
<td>$752</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-$500</td>
<td>$0</td>
<td>$428</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0</td>
<td>$185</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Cash Flow for Levered PE Investment

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$500</td>
<td>$0</td>
<td>$0</td>
<td>$500</td>
</tr>
<tr>
<td>$554</td>
<td>$845</td>
<td>$216</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-$500</td>
<td>$0</td>
<td>$585</td>
<td></td>
<td>$18</td>
</tr>
<tr>
<td></td>
<td>$0</td>
<td>$405</td>
<td></td>
<td>$180</td>
</tr>
</tbody>
</table>

Panel C: PV using Rubinstein CAPM

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Levered Market</td>
<td>-$500.00</td>
<td>$0.00</td>
<td>$0.00</td>
<td>$500.00</td>
<td>$0.00</td>
</tr>
<tr>
<td>Levered PE</td>
<td>-$500.00</td>
<td>$0.00</td>
<td>$542.54</td>
<td>$65.14</td>
<td>$107.68</td>
</tr>
</tbody>
</table>